Pari-GP reference card

(PARI-GP version 2.15.3)

Note: optional arguments are surrounded by braces {}.
To start the calculator, type its name in the terminal: `gp`
To exit `gp`, type `quit`, `\q`, or `<C-D>` at prompt.

Help

describe function	`?`*function*
extended description	`??`*keyword*
list of relevant help topics	`???`*pattern*
name of GP-1.39 function f in GP-2.*	`whatnow`(f)

Input/Output

previous result, the result before	`%`, `%\``, `%\`\``, etc.
n-th result since startup	`%`n
separate multiple statements on line	`;`
extend statement on additional lines	`\`
extend statements on several lines	`{`seq_1`;` seq_2`;}`
comment	`/* ... */`
one-line comment, rest of line ignored	`\\ ...`

Metacommands & Defaults

set default d to *val*	`default`({d}, {*val*})
toggle timer on/off	`#`
print time for last result	`##`
print defaults	`\d`
set debug level to n	`\g` n
set memory debug level to n	`\gm` n
set n significant digits / bits	`\p` n, `\pb` n
set n terms in series	`\ps` n
quit GP	`\q`
print the list of PARI types	`\t`
print the list of user-defined functions	`\u`
read file into GP	`\r` *filename*
set debuglevel for domain D to n	`setdebug`(D, n)

Debugger / break loop

get out of break loop	`break` or `<C-D>`
go up/down n frames	`dbg_up`({n}), `dbg_down`
set break point	`breakpoint`()
examine object o	`dbg_x`(o)
current error data	`dbg_err`()
number of objects on heap and their size	`getheap`()
total size of objects on PARI stack	`getstack`()

PARI Types & Input Formats

`t_INT`. Integers; hex, binary	±31; ±`0x1F`, ±`0b101`
`t_REAL`. Reals	±3.14, 6.022 `E23`
`t_INTMOD`. Integers modulo m	`Mod`(n, m)
`t_FRAC`. Rational Numbers	n/m
`t_FFELT`. Elt in finite field $\mathbf{F}_q$	`ffgen`(q,'t)
`t_COMPLEX`. Complex Numbers	$x + y *$ `I`
`t_PADIC`. p-adic Numbers	$x +$ `O`(p`^`k)
`t_QUAD`. Quadratic Numbers	$x + y *$ `quadgen`(D,{'w})
`t_POLMOD`. Polynomials modulo g	`Mod`(f, g)
`t_POL`. Polynomials	$a * x$`^`$n + \cdots + b$
`t_SER`. Power Series	$f +$ `O`(x`^`k)
`t_RFRAC`. Rational Functions	f/g
`t_QFB`. Binary quadratic form	`Qfb`(a, b, c)
`t_VEC`/`t_COL`. Row/Column Vectors	[x, y, z], [x, y, z]~
`t_VEC` integer range	`[1..10]`
`t_VECSMALL`. Vector of small ints	`Vecsmall`([x, y, z])
`t_MAT`. Matrices	[a, b;c, d]
`t_LIST`. Lists	`List`([x, y, z])
`t_STR`. Strings	`"abc"`
`t_INFINITY`. $\pm\infty$	`+oo, -oo`

Reserved Variable Names

$\pi \approx 3.14$, $\gamma \approx 0.57$, $C \approx 0.91$, $I = \sqrt{-1}$	`Pi`, `Euler`, `Catalan`, `I`
Landau's big-oh notation	`O`

Information about an Object, Precision

PARI type of object x	`type`(x)
length of x / size of x in memory	`#`x, `sizebyte`(x)
real precision / bit precision of x	`precision`(x), `bitprecision`(x)
p-adic, series prec. of x	`padicprec`(x, p), `serprec`(x, v)
current dynamic precision	`getlocalprec`, `getlocalbitprec`

Operators

basic operations	`+`, `-` , `*`, `/`, `^`, `sqr`
`i`←`i+1`, `i`←`i-1`, `i`←`i*j`, ...	`i++`, `i--`, `i*=j`,...
Euclidean quotient, remainder	x`\/`y, x`\`y, x`%`y, `divrem`(x, y)
shift x left or right n bits	x`<<`n, x`>>`n or `shift`($x, \pm n$)
multiply by 2^n	`shiftmul`(x, n)
comparison operators	`<=`, `<`, `>=`, `>`, `==`, `!=`, `===`, `lex`, `cmp`
boolean operators (or, and, not)	`||`, `&&`, `!`
bit operations	`bitand`, `bitneg`, `bitor`, `bitxor`, `bitnegimply`
maximum/minimum of x and y	`max`(x, y), `min`(x, y)
sign of x (gives $-1, 0, 1$)	`sign`(x)
binary exponent of x	`exponent`(x)
derivative of f, 2nd derivative, etc.	f`'`, f`''`, ...
differential operator	`diffop`(f, v, d, {$n = 1$})
quote operator (formal variable)	`'x`
assignment	`x =` *value*
simultaneous assignment $x \leftarrow v[1]$, $y \leftarrow v[2]$	`[x,y] = v`

Select Components

Caveat: components start at index $n = 1$.

n-th component of x	`component`(x, n)
n-th component of vector/list x	x[n]
components $a, a+1, \ldots, b$ of vector x	x[a..b]
(m, n)-th component of matrix x	x[m, n]
row m or column n of matrix x	x[m,], x[,n]
numerator/denominator of x	`numerator`(x), `denominator`(x)

Random Numbers

random integer/prime in $[0, N[$	`random`(N), `randomprime`(N)
get/set random seed	`getrand`, `setrand`(s)

Conversions

to vector, matrix, vec. of small ints	`Col`/`Vec`,`Mat`,`Vecsmall`
to list, set, map, string	`List`, `Set`, `Map`, `Str`
create (x mod y)	`Mod`(x, y)
make x a polynomial of v	`Pol`(x, {v})
variants of `Pol` *et al.*, in reverse order	`Polrev`, `Vecrev`, `Colrev`
make x a power series of v	`Ser`(x, {v})
convert x to simplest possible type	`simplify`(x)
object x with real precision n	`precision`(x, n)
object x with bit precision n	`bitprecision`(x, n)
set precision to p digits in dynamic scope	`localprec`(p)
set precision to p bits in dynamic scope	`localbitprec`(p)

Character strings

convert to TeX representation	`strtex`(x)
string from bytes / from format+args	`strchr`, `strprintf`
split string / join strings	`strsplit`, `strjoin`
convert time t ms. to h, m, s, ms format	`strtime`(t)

Conjugates and Lifts

conjugate of a number x	`conj`(x)
norm of x, product with conjugate	`norm`(x)
L^p norm of x (L^∞ if no p)	`normlp`(x, {p})
square of L^2 norm of x	`norml2`(x)
lift of x from Mods and p-adics	`lift`, `centerlift`(x)
recursive lift	`liftall`
lift all `t_INT` and `t_PADIC` (→`t_INT`)	`liftint`
lift all `t_POLMOD` (→`t_POL`)	`liftpol`

Lists, Sets & Maps

Sets (= row vector with strictly increasing entries w.r.t. `cmp`)

intersection of sets x and y	`setintersect`(x, y)
set of elements in x not belonging to y	`setminus`(x, y)
symmetric difference $x \Delta y$	`setdelta`(x, y)
union of sets x and y	`setunion`(x, y)
does y belong to the set x	`setsearch`(x, y, {*flag*})
set of all $f(x, y)$, $x \in X$, $y \in Y$	`setbinop`(f, X, Y)
is x a set ?	`setisset`(x)

Lists. create empty list: $L =$ `List`()

append x to list L	`listput`(L, x, {i})
remove i-th component from list L	`listpop`(L, {i})
insert x in list L at position i	`listinsert`(L, x, i)
sort the list L in place	`listsort`(L, {*flag*})

Maps. create empty dictionary: $M =$ `Map`()

attach value v to key k	`mapput`(M, k, v)
recover value attach to key k or error	`mapget`(M, k)
is key k in the dict? (set v to $M(k)$)	`mapisdefined`(M, k, {&v})
remove k from map domain	`mapdelete`(M, k)

GP Programming

User functions and closures

x, y are formal parameters; y defaults to `Pi` if parameter omitted; z, t are local variables (lexical scope), z initialized to 1.

`fun(x, y=Pi) = my(z=1, t);` *seq*

`fun = (x, y=Pi) -> my(z=1, t);` *seq*

attach help message h to s	`addhelp`(s, h)
undefine symbol s (also kills help)	`kill`(s)

Control Statements (X: formal parameter in expression *seq*)

if $a \neq 0$, evaluate seq_1, else seq_2	`if`(a, {seq_1}, {seq_2})
eval. *seq* for $a \le X \le b$	`for`($X = a, b, seq$)
... for $X \in v$	`foreach`(v, X, seq)
... for primes $a \le X \le b$	`forprime`($X = a, b, seq$)
... for primes $\equiv a \pmod q$	`forprimestep`($X = a, b, q, seq$)
... for composites $a \le X \le b$	`forcomposite`($X = a, b, seq$)
... for $a \le X \le b$ stepping s	`forstep`($X = a, b, s, seq$)
... for X dividing n	`fordiv`(n, X, seq)
... $X = [n, factor(n)]$, $a \le n \le b$	`forfactored`($X = a, b, seq$)
... as above, n squarefree	`forsquarefree`($X = a, b, seq$)
... $X = [d, factor(d)]$, $d \mid n$	`fordivfactored`(n, X, seq)
multivariable `for`, lex ordering	`forvec`($X = v, seq$)

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loop over partitions of n	`forpart`$(p=n, seq)$
... permutations of S	`forperm`(S, p, seq)
... subsets of $\{1,\ldots,n\}$	`forsubset`(n, p, seq)
... k-subsets of $\{1,\ldots,n\}$	`forsubset`$([n,k], p, seq)$
... vectors v, $q(v) \le B$; $q > 0$	`forqfvec`(v, q, b, seq)
... $H < G$ finite abelian group	`forsubgroup`$(H = G)$
evaluate *seq* until $a \neq 0$	`until`(a, seq)
while $a \neq 0$, evaluate *seq*	`while`(a, seq)
exit n innermost enclosing loops	`break`$(\{n\})$
start new iteration of n-th enclosing loop	`next`$(\{n\})$
return x from current subroutine	`return`$(\{x\})$

Exceptions, warnings

raise an exception / warning	`error(), warning()`
type of error message E	`errname`(E)
try seq_1, evaluate seq_2 on error	`iferr`(seq_1, E, seq_2)

Functions with closure arguments / results

number of arguments of f	`arity`(f)
select from v according to f	`select`(f, v)
apply f to all entries in v	`apply`(f, v)
evaluate $f(a_1, \ldots, a_n)$	`call`(f, a)
evaluate $f(\ldots f(f(a_1, a_2), a_3) \ldots, a_n)$	`fold`(f, a)
calling function as closure	`self()`

Sums & Products

sum $X = a$ to $X = b$, initialized at x	`sum`$(X = a, b, expr, \{x\})$
sum entries of vector v	`vecsum`(v)
product of all vector entries	`vecprod`(v)
sum *expr* over divisors of n	`sumdiv`$(n, X, expr)$
... assuming *expr* multiplicative	`sumdivmult`$(n, X, expr)$
product $a \le X \le b$, initialized at x	`prod`$(X = a, b, expr, \{x\})$
product over primes $a \le X \le b$	`prodeuler`$(X = a, b, expr)$

Sorting

sort x by k-th component	`vecsort`$(x, \{k\}, \{fl = 0\})$
min. m of x ($m = x[i]$), max.	`vecmin`$(x, \{\&i\})$, `vecmax`
does y belong to x, sorted wrt. f	`vecsearch`$(x, y, \{f\})$
$\prod g^x \to$ factorization ($\Rightarrow$ sorted, unique g)	`matreduce`(m)

Input/Output

print with/without `\n`, TEX format	`print`, `print1`, `printtex`
pretty print matrix	`printp`
print fields with separator	`printsep`$(sep, \ldots)$,, `printsep1`
formatted printing	`printf()`
write *args* to file	`write`, `write1`, `writetex`$(file, args)$
write x in binary format	`writebin`$(file, x)$
read file into GP	`read`$(\{file\})$
... return as vector of lines	`readvec`$(\{file\})$
... return as vector of strings	`readstr`$(\{file\})$
read a string from keyboard	`input()`

Files and file descriptors

File descriptors allow efficient small consecutive reads or writes from or to a given file. The argument n below is always a descriptor, attached to a file in `r`(ead), `w`(rite) or `a`(ppend) mode.

get descriptor n for file *path* in given *mode*	`fileopen`$(path, mode)$
... from shell *cmd* output (pipe)	`fileextern`(cmd)
close descriptor	`fileclose`(n)
commit pending write operations	`fileflush`(n)
read logical line from file	`fileread`(n)
... raw line from file	`filereadstr`(n)
write s`\n` to file	`filewrite`(n, s)
... write s to file	`filewrite1`(n, s)

Timers

CPU time in *ms* and reset timer	`gettime()`
CPU time in *ms* since gp startup	`getabstime()`
time in *ms* since UNIX Epoch	`getwalltime()`
timeout command after s seconds	`alarm`$(s, expr)$

Interface with system

allocates a new stack of s bytes	`allocatemem`$(\{s\})$
alias *old* to *new*	`alias`(new, old)
install function from library	`install`$(f, code, \{gpf\}, \{lib\})$
execute system command a	`system`(a)
... and feed result to GP	`extern`(a)
... returning GP string	`externstr`(a)
get `$VAR` from environment	`getenv("VAR")`
expand env. variable in string	`strexpand`(x)

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables (use `export` for this) and must be free of side effects. Enabled if threading engine is not *single* in gp header.

evaluate f on $x[1], \ldots, x[n]$	`parapply`(f, x)
evaluate closures $f[1], \ldots, f[n]$	`pareval`(f)
as `select`	`parselect`$(f, A, \{flag\})$
as `sum`	`parsum`$(i = a, b, expr)$
as `vector`	`parvector`$(n, i, \{expr\})$
eval f for $i = a, \ldots, b$	`parfor`$(i = a, \{b\}, f, \{r\}, \{f_2\})$
... for each element x in v	`parforeach`$(v, x, f, \{r\}, \{f_2\})$
... for p prime in $[a, b]$	`parforprime`$(p = a, \{b\}, f, \{r\}, \{f_2\})$
... for $p = a$ mod q	`parforprimestep`$(p = a, \{b\}, q, f, \{r\}, \{f_2\})$
... multivariate	`parforvec`$(X = v, f, \{r\}, \{f_2\}, \{flag\})$
export x to parallel world	`export`(x)
... all dynamic variables	`exportall()`
frees exported value x	`unexport`(x)
... all exported values	`unexportall()`

Linear Algebra

dimensions of matrix x	`matsize`(x)
multiply two matrices	x `*` y
... assuming result is diagonal	`matmultodiagonal`(x, y)
concatenation of x and y	`concat`$(x, \{y\})$
extract components of x	`vecextract`$(x, y, \{z\})$
transpose of vector or matrix x	x`~`, `mattranspose`(x)
adjoint of the matrix x	`matadjoint`(x)
eigenvectors/values of matrix x	`mateigen`(x)
characteristic/minimal polynomial of x	`charpoly`(x), `minpoly`(x)
trace/determinant of matrix x	`trace`(x), `matdet`(x)
permanent of matrix x	`matpermanent`(x)
Frobenius form of x	`matfrobenius`(x)
QR decomposition	`matqr`(x)
apply `matqr`'s transform to v	`mathouseholder`(Q, v)

Constructors & Special Matrices

$\{g(x) : x \in v$ s.t. $f(x)\}$	`[g(x) | x <- v, f(x)]`
$\{x : x \in v$ s.t. $f(x)\}$	`[x | x <- v, f(x)]`
$\{g(x) : x \in v\}$	`[g(x) | x <- v]`
row vec. of *expr* eval'ed at $1 \le i \le n$	`vector`$(n, \{i\}, \{expr\})$
col. vec. of *expr* eval'ed at $1 \le i \le n$	`vectorv`$(n, \{i\}, \{expr\})$
vector of small ints	`vectorsmall`$(n, \{i\}, \{expr\})$
$[c, c \cdot x, \ldots, c \cdot x^n]$	`powers`$(x, n, \{c = 1\})$
$[1, 2^x, \ldots, n^x]$	`dirpowers`(n, x)
matrix $1 \le i \le m$, $1 \le j \le n$	`matrix`$(m, n, \{i\}, \{j\}, \{expr\})$
define matrix by blocks	`matconcat`(B)
diagonal matrix with diagonal x	`matdiagonal`(x)
is x diagonal?	`matisdiagonal`(x)
$x \cdot$ `matdiagonal`(d)	`matmuldiagonal`(x, d)
$n \times n$ identity matrix	`matid`(n)
Hessenberg form of square matrix x	`mathess`(x)
$n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$	`mathilbert`(n)
$n \times n$ Pascal triangle	`matpascal`$(n - 1)$
companion matrix to polynomial x	`matcompanion`(x)
Sylvester matrix of x and y	`polsylvestermatrix`(x, y)

Gaussian elimination

kernel of matrix x	`matker`$(x, \{flag\})$
intersection of column spaces of x and y	`matintersect`(x, y)
solve $MX = B$ (M invertible)	`matsolve`(M, B)
one sol of $M * X = B$	`matinverseimage`(M, B)
basis for image of matrix x	`matimage`(x)
columns of x *not* in `matimage`	`matimagecompl`(x)
supplement columns of x to get basis	`matsupplement`(x)
rows, cols to extract invertible matrix	`matindexrank`(x)
rank of the matrix x	`matrank`(x)
solve $MX = B$ mod D	`matsolvemod`(M, D, B)
image mod D	`matimagemod`(M, D)
kernel mod D	`matkermod`(M, D)
inverse mod D	`matinvmod`(M, D)
determinant mod D	`matdetmod`(M, D)

Lattices & Quadratic Forms

Quadratic forms

evaluate ${}^t xQy$	`qfeval`$(\{Q = id\}, x, y)$
evaluate ${}^t xQx$	`qfeval`$(\{Q = id\}, x)$
signature of quad form ${}^t y * x * y$	`qfsign`(x)
decomp into squares of ${}^t y * x * y$	`qfgaussred`(x)
eigenvalues/vectors for real symmetric x	`qfjacobi`(x)

HNF and SNF

upper triangular Hermite Normal Form	`mathnf`(x)
HNF of x where d is a multiple of $\det(x)$	`mathnfmod`(x, d)
multiple of $\det(x)$	`matdetint`(x)
HNF of $(x \mid$ `diagonal`$(D))$	`mathnfmodid`(x, D)
elementary divisors of x	`matsnf`(x)
q-rank from elementary divisors	`snfrank`(v, q)
elementary divisors of $\mathbf{Z}[a]/(f'(a))$	`poldiscreduced`(f)
integer kernel of x	`matkerint`(x)
$\mathbf{Z}$-module $\leftrightarrow$ $\mathbf{Q}$-vector space	`matrixqz`(x, p)

Lattices

LLL-algorithm applied to columns of x	`qflll`$(x, \{flag\})$
... for Gram matrix of lattice	`qflllgram`$(x, \{flag\})$
find up to m sols of `qfnorm`$(x, y) \le b$	`qfminim`(x, b, m)
v, $v[i]$:=number of y s.t. `qfnorm`$(x, y) = i$	`qfrep`$(x, B, \{flag\})$
perfection rank of x	`qfperfection`(x)
find isomorphism between q and Q	`qfisom`(q, Q)
precompute for isomorphism test with q	`qfisominit`(q)
automorphism group of q	`qfauto`(q)

convert `qfauto` for GAP/Magma	`qfautoexport(G, {flag})`
orbits of V under $G \subset \mathrm{GL}(V)$	`qforbits(G, V)`

Polynomials & Rational Functions

all defined polynomial variables	`variables()`
get var. of highest priority (higher than v)	`varhigher(name, {v})`
... of lowest priority (lower than v)	`varlower(name, {v})`

Coefficients, variables and basic operators

degree of f	`poldegree(f)`
coef. of degree n of f, leading coef.	`polcoef(f, n)`, `pollead`
main variable / all variables in f	`variable(f)`, `variables(f)`
replace x by y in f	`subst(f, x, y)`
evaluate f replacing vars by their value	`eval(f)`
replace polynomial expr. $T(x)$ by y in f	`substpol(f, T, y)`
replace $x_1, \ldots, x_n$ by $y_1, \ldots, y_n$ in f	`substvec(f, x, y)`
$f \in A[x]$; reciprocal polynomial $x^{\deg f} f\left(\frac{1}{x}\right)$	`polrecip(f)`
gcd of coefficients of f	`content(f)`
derivative of f w.r.t. x	`deriv(f, {x})`
... n-th derivative of f	`derivn(f, n, {x})`
formal integral of f w.r.t. x	`intformal(f, {x})`
formal sum of f w.r.t. x	`sumformal(f, {x})`

Constructors & Special Polynomials

interpolation polynomial at $(x[1], y[1]), \ldots, (x[n], y[n])$, evaluated at t, with error estimate e	`polinterpolate(x, {y}, {t}, {&e})`
T_n/U_n, H_n	`polchebyshev(n)`, `polhermite(n)`
P_n, $L_n^{(\alpha)}$	`pollegendre(n)`, `pollaguerre(n, a)`
n-th cyclotomic polynomial Φ_n	`polcyclo(n)`
return n if $f = \Phi_n$, else 0	`poliscyclo(f)`
is f a product of cyclotomic polynomials?	`poliscycloprod(f)`
Zagier's polynomial of index (n, m)	`polzagier(n, m)`

Resultant, elimination

discriminant of polynomial f	`poldisc(f)`
find factors of `poldisc(f)`	`poldiscfactors(f)`
resultant $R = \mathrm{Res}_v(f, g)$	`polresultant(f, g, {v})`
$[u, v, R]$, $xu + yv = \mathrm{Res}_v(f, g)$	`polresultantext(x, y, {v})`
solve Thue equation $f(x, y) = a$	`thue(t, a, {sol})`
initialize t for Thue equation solver	`thueinit(f)`

Roots and Factorization (Complex/Real)

complex roots of f	`polroots(f)`
bound complex roots of f	`polrootsbound(f)`
number of real roots of f (in $[a, b]$)	`polsturm(f, {[a, b]})`
real roots of f (in $[a, b]$)	`polrootsreal(f, {[a, b]})`
complex embeddings of `t_POLMOD` z	`conjvec(z)`

Roots and Factorization (Finite fields)

factor f mod p, roots	`factormod(f, p)`, `polrootsmod`
factor f over $\mathbf{F}_p[x]/(T)$, roots	`factormod(f, [T, p])`, `polrootsmod`
squarefree factorization of f in $\mathbf{F}_q[x]$	`factormodSQF(f, {D})`
distinct degree factorization of f in $\mathbf{F}_q[x]$	`factormodDDF(f, {D})`
factor n-th cyclotomic pol. Φ_n mod p	`factormodcyclo(n, p)`

Roots and Factorization (p-adic fields)

factor f over $\mathbf{Q}_p$, roots	`factorpadic(f, p, r)`, `polrootspadic`
p-adic root of f congruent to a mod p	`padicappr(f, a)`
Newton polygon of f for prime p	`newtonpoly(f, p)`
Hensel lift $A/\mathrm{lc}(A) = \prod_i B[i]$ mod p^e	`polhensellift(A, B, p, e)`
$T = \prod(x - z_i) \mapsto \prod(x - \omega(z_i))] \in \mathbf{Z}_p[x]$	`polteichmuller(T, p, e)`
extensions of $\mathbf{Q}_p$ of degree N	`padicfields(p, N)`

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Roots and Factorization (Miscellaneous)

symmetric powers of roots of f up to n	`polsym(f, n)`
Graeffe transform of f, $g(x^2) = f(x)f(-x)$	`polgraeffe(f)`
factor f over coefficient field	`factor(f)`
cyclotomic factors of $f \in \mathbf{Q}[X]$	`polcyclofactors(f)`

Finite Fields

A finite field is encoded by any element (`t_FFELT`).

find irreducible $T \in \mathbf{F}_p[x]$, $\deg T = n$	`ffinit(p, n, {x})`
Create t in $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$	`t = ffgen(T,'t)`
... indirectly, with implicit T	`t = ffgen(q,'t); T = t.mod`
map m from $\mathbf{F}_q \ni a$ to $\mathbf{F}_{q^k} \ni b$	`m = ffembed(a, b)`
build $K = \mathbf{F}_q[x]/(P)$ extending $\mathbf{F}_q \ni a$,	`ffextend(a, P)`
evaluate map m on x	`ffmap(m, x)`
inverse map of m	`ffinvmap(m)`
compose maps $m \circ n$	`ffcompomap(m, n)`
x as polmod over codomain of map m	`ffmaprel(m, x)`
F^n over $\mathbf{F}_q \ni a$	`fffrobenius(a, n)`
$\#\{$monic irred. $T \in \mathbf{F}_q[x], \deg T = n\}$	`ffnbirred(q, n)`

Formal & p-adic Series

truncate power series or p-adic number	`truncate(x)`
valuation of x at p	`valuation(x, p)`

Dirichlet and Power Series

Taylor expansion around 0 of f w.r.t. x	`taylor(f, x)`
Laurent series of closure F up to x^k	`laurentseries(f, k)`
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	`serconvol(a, b)`
$f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$	`serlaplace(f)`
reverse power series F so $F(f(x)) = x$	`serreverse(f)`
remove terms of degree $< n$ in f	`serchop(f, n)`
Dirichlet series multiplication / division	`dirmul`, `dirdiv(x, y)`
Dirichlet Euler product (b terms)	`direuler(p = a, b, expr)`

Transcendental and p-adic Functions

real, imaginary part of x	`real(x)`, `imag(x)`
absolute value, argument of x	`abs(x)`, `arg(x)`
square/nth root of x	`sqrt(x)`, `sqrtn(x, n, {&z})`
all n-th roots of 1	`rootsof1(n)`
FFT of $[f_0, \ldots, f_{n-1}]$	w `= fftinit(n)`, `fft/fftinv(w, f)`
trig functions	`sin`, `cos`, `tan`, `cotan`, `sinc`
inverse trig functions	`asin`, `acos`, `atan`
hyperbolic functions	`sinh`, `cosh`, `tanh`, `cotanh`
inverse hyperbolic functions	`asinh`, `acosh`, `atanh`
$\log(x)$, $\log(1 + x)$, e^x, $e^x - 1$	`log`, `log1p`, `exp`, `expm1`
Euler Γ function, $\log \Gamma$, Γ'/Γ	`gamma`, `lngamma`, `psi`
half-integer gamma function $\Gamma(n + 1/2)$	`gammah(n)`
Riemann's zeta $\zeta(s) = \sum n^{-s}$	`zeta(s)`
$\sum_{1 \le n \le N} n^s$	`dirpowerssum(N, s)`
Hurwitz's $\zeta(s, x) = \sum (n + x)^{-s}$	`zetahurwitz(s, x)`
Lerch $\Phi(z, s, x) = \sum z^n (n + x)^{-s}$	`lerchphi(z, s, x)`
Lerch $L(s, x, t) = \Phi(e^{2i\pi t}, s, x)$	`lerchzeta(s, x, t)`
multiple zeta value (MZV), $\zeta(s_1, \ldots, s_k)$	`zetamult(s, {T})`
all MZVs for weight $\sum s_i = n$	`zetamultall(n)`
convert MZV id to $[s_1, \ldots, s_k]$	`zetamultconvert(f, {flag})`
MZV dual sequence	`zetamultdual(s)`
multiple polylog $Li_{s_1, \ldots s_k}(z_1, \ldots, z_k)$	`polylogmult(s, z)`
incomplete Γ function ($y = \Gamma(s)$)	`incgam(s, x, {y})`
complementary incomplete Γ	`incgamc(s, x)`
$\int_x^\infty e^{-t}\, dt/t$, $\quad (2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$	`eint1`, `erfc`
elliptic integral of 1st and 2nd kind	`ellK(k)`, `ellE(k)`
dilogarithm of x	`dilog(x)`
m-th polylogarithm of x	`polylog(m, x, {flag})`
U-confluent hypergeometric function	`hyperu(a, b, u)`
Hypergeometric ${}_pF_q(A, B; z)$	`hypergeom(A, B, z)`
Bessel $J_n(x)$, $J_{n+1/2}(x)$	`besselj(n, x)`, `besseljh(n, x)`
Bessel I_ν, K_ν, H_ν^1, H_ν^2, Y_ν	`(bessel)i`, `k`, `h1`, `h2`, `y`
k-th zero of $J_\nu(x)$	`besseljzero(nu, {k = 1})`
k-th zero of $Y_\nu(x)$	`besselyzero(nu, {k = 1})`
Airy functions $A_i(x)$, $B_i(x)$	`airy(x)`
Lambert W: x s.t. $xe^x = y$	`lambertw(y)`
Teichmuller character of p-adic x	`teichmuller(x)`

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ($a \in \mathbf{C}$, regular) or $\pm$`oo` (decreasing at least as x^{-2}) or

$(x - a)^{-\alpha}$ singularity	$[a, \alpha]$
exponential decrease $e^{-\alpha\lvert x\rvert}$	$[\pm\infty, \alpha]$, $\alpha > 0$
slow decrease $\lvert x\rvert^\alpha$	$\ldots \alpha < -1$
oscillating as $\cos(kx)$)	$\alpha = k$`I`, $k > 0$
oscillating as $\sin(kx)$)	$\alpha = -k$`I`, $k > 0$
numerical integration	`intnum(x = a, b, f, {T})`
weights T for `intnum`	`intnuminit(a, b, {m})`
weights T incl. kernel K	`intfuncinit(t = a, b, K, {m})`
integrate $(2i\pi)^{-1} f$ on circle $\lvert z - a\rvert = R$	`intcirc(x = a, R, f, {T})`

Other integration methods

n-point Gauss-Legendre	`intnumgauss(x = a, b, f, {n})`
weights for n-point Gauss-Legendre	`intnumgaussinit({n})`
quasi-periodic function, period $2H$	`intnumosc(x = a, f, H)`
Romberg (low accuracy)	`intnumromb(x = a, b, f, {flag})`

Numerical summation

sum of series $f(n)$, $n \ge a$ (low accuracy)	`suminf(n = a, expr)`
sum of alternating/positive series	`sumalt`, `sumpos`
sum of series using Euler-Maclaurin	`sumnum(n = a, f, {T})`
... Sidi summation	`sumnumsidi(n = a, f)`
$\sum_{n \ge a} F(n)$, F rational function	`sumnumrat(F, a)`
... $\sum_{p \ge a} F(p^s)$	`sumeulerrat(F, {s = 1}, {a = 2})`
weights for `sumnum`, a as in DE	`sumnuminit({∞, a})`
sum of series by Monien summation	`sumnummonien(n = a, f, {T})`
weights for `sumnummonien`	`sumnummonieninit({∞, a})`
sum of series using Abel-Plana	`sumnumap(n = a, f, {T})`
weights for `sumnumap`, a as in DE	`sumnumapinit({∞, a})`
sum of series using Lagrange	`sumnumlagrange(n = a, f, {T})`
weights for `sumnumlagrange`	`sumnumlagrangeinit`

Products

product $a \le X \le b$, initialized at x	`prod(X = a, b, expr, {x})`
product over primes $a \le X \le b$	`prodeuler(X = a, b, expr)`
infinite product $a \le X \le \infty$	`prodinf(X = a, expr)`
$\prod_{n \ge a} F(n)$, F rational function	`prodnumrat(F, a)`
$\prod_{p \ge a} F(p^s)$	`prodeulerrat(F, {s = 1}, {a = 2})`

Other numerical methods

real root of f in $[a,b]$; bracketed root	`solve`$(X=a,b,f)$
... interval splitting, step s	`solvestep`$(X=a,b,s,f,\{flag=0\})$
limit of $f(t)$, $t\to\infty$	`limitnum(f, {alpha})`
asymptotic expansion of f (rational)	`asympnum(f, {alpha})`
... $N+1$ terms as floats	`asympnumraw(f, N, {alpha})`
numerical derivation w.r.t x: $f'(a)$	`derivnum`$(x=a,f)$
evaluate continued fraction F at t	`contfraceval`$(F,t,\{L\})$
power series to cont. fraction (L terms)	`contfracinit`$(S,\{L\})$
Padé approximant (deg. denom. $\leq B$)	`bestapprPade`$(S,\{B\})$

Elementary Arithmetic Functions

vector of binary digits of $\lvert x\rvert$	`binary`(x)
bit number n of integer x	`bittest`(x,n)
Hamming weight of integer x	`hammingweight`(x)
digits of integer x in base B	`digits`$(x,\{B=10\})$
sum of digits of integer x in base B	`sumdigits`$(x,\{B=10\})$
integer from digits	`fromdigits`$(v,\{B=10\})$
ceiling/floor/fractional part	`ceil`, `floor`, `frac`
round x to nearest integer	`round`$(x,\{\&e\})$
truncate x	`truncate`$(x,\{\&e\})$
gcd/LCM of x and y	`gcd`(x,y), `lcm`(x,y)
gcd of entries of a vector/matrix	`content`(x)

Primes and Factorization

extra prime table	`addprimes()`
add primes in v to prime table	`addprimes`(v)
remove primes from prime table	`removeprimes`(v)
Chebyshev $\pi(x)$, n-th prime p_n	`primepi`(x), `prime`(n)
vector of first n primes	`primes`(n)
smallest prime $\geq x$	`nextprime`(x)
largest prime $\leq x$	`precprime`(x)
factorization of x	`factor`$(x,\{lim\})$
... selecting specific algorithms	`factorint`$(x,\{flag=0\})$
$n=df^2$, d squarefree/fundamental	`core`$(n,\{fl\})$, `coredisc`
certificate for (prime) N	`primecert`(N)
verifies a certificate c	`primecertisvalid`(c)
convert certificate to Magma/PRIMO	`primecertexport`
recover x from its factorization	`factorback`$(f,\{e\})$
$x\in\mathbf{Z}$, $\lvert x\rvert\leq X$, $\gcd(N,P(x))\geq N$	`zncoppersmith`$(P,N,X,\{B\})$
divisors of N in residue class r mod s	`divisorslenstra`(N,r,s)

Divisors and multiplicative functions

number of prime divisors $\omega(n)$ / $\Omega(n)$	`omega`(n), `bigomega`
divisors of n / number of divisors $\tau(n)$	`divisors`(n), `numdiv`
sum of (k-th powers of) divisors of n	`sigma`$(n,\{k\})$
Möbius μ-function	`moebius`(x)
Ramanujan's τ-function	`ramanujantau`(x)

Combinatorics

factorial of x	`x!` or `factorial`(x)
binomial coefficient $\binom{x}{k}$	`binomial`$(x,\{k\})$
Bernoulli number B_n as real/rational	`bernreal`(n), `bernfrac`
$[B_0,B_2,\ldots B_{2k}]$	`bernvec`(k)
Bernoulli polynomial $B_n(x)$	`bernpol`$(n,\{x\})$
Euler numbers	`eulerfrac`, `eulerreal`, `eulervec`
Euler polynomial $E_n(x)$	`eulerpol`$(n,\{x\})$
Eulerian polynomial $A_n(x)$	`eulerianpol`
Fibonacci number F_n	`fibonacci`(n)
Harmonic number $H_{n,r}=1^{-r}+\ldots+n^{-r}$	`harmonic`(n,r)
Stirling numbers $s(n,k)$ and $S(n,k)$	`stirling`$(n,k,\{flag\})$

Pari-GP reference card

(PARI-GP version 2.15.3)

number of partitions of n	`numbpart`(n)
k-th permutation on n letters	`numtoperm`(n,k)
... index k of permutation v	`permtonum`(v)
order of permutation p	`permorder`(p)
signature of permutation p	`permsign`(p)
cyclic decomposition of permutation p	`permcycles`(p)

Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, $\mathbf{F}_q^*$

Euler ϕ-function	`eulerphi`(x)
multiplicative order of x (divides o)	`znorder`$(x,\{o\})$, `fforder`
primitive root mod q / x`.mod`	`znprimroot`(q), `ffprimroot`(x)
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	`znstar`(n)
discrete logarithm of x in base g	`znlog`$(x,g,\{o\})$, `fflog`
Kronecker-Legendre symbol $(\frac{x}{y})$	`kronecker`(x,y)
quadratic Hilbert symbol (at p)	`hilbert`$(x,y,\{p\})$

Euclidean algorithm, continued fractions

CRT: solve $z\equiv x$ and $z\equiv y$	`chinese`(x,y)
minimal u,v so $xu+yv=\gcd(x,y)$	`gcdext`(x,y)
half-gcd algorithm	`halfgcd`(x,y)
continued fraction of x	`contfrac`$(x,\{b\},\{lmax\})$
last convergent of continued fraction x	`contfracpnqn`(x)
rational approximation to x (den. $\leq B$)	`bestappr`$(x,\{B\})$
recognize $x\in\mathbf{C}$ as polmod mod $T\in\mathbf{Z}[X]$	`bestapprnf`(x,T)

Miscellaneous

integer square / n-th root of x	`sqrtint`(x), `sqrtnint`(x,n)
largest integer e s.t. $b^e\leq b$, $e=\lfloor\log_b(x)\rfloor$	`logint`$(x,b,\{\&z\})$

Characters

Let $cyc=[d_1,\ldots,d_k]$ represent an abelian group $G=\oplus(\mathbf{Z}/d_j\mathbf{Z})\cdot g_j$ or any structure G affording a `.cyc` method; e.g. `znstar`$(q,1)$ for Dirichlet characters. A character χ is coded by $[c_1,\ldots,c_k]$ such that $\chi(g_j)=e(n_j/d_j)$.

$\chi\cdot\psi$; χ^{-1}; $\chi\cdot\psi^{-1}$; χ^k	`charmul`, `charconj`, `chardiv`, `charpow`
order of χ	`charorder`(cyc,χ)
kernel of χ	`charker`(cyc,χ)
$\chi(x)$, G a GP group structure	`chareval`$(G,\chi,x,\{z\})$
Galois orbits of characters	`chargalois`(G)

Dirichlet Characters

initialize $G=(\mathbf{Z}/q\mathbf{Z})^*$	`G = znstar`$(q,1)$
convert datum D to $[G,\chi]$	`znchar`(D)
is χ odd?	`zncharisodd`(G,χ)
real $\chi\to$ Kronecker symbol $(D/.)$	`znchartokronecker`(G,χ)
conductor of χ	`zncharconductor`(G,χ)
$[G_0,\chi_0]$ primitive attached to χ	`znchartoprimitive`(G,χ)
induce $\chi\in\hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$	`zncharinduce`(G,χ,N)
χ_p	`znchardecompose`(G,χ,p)
$\prod_{p\mid(Q,N)}\chi_p$	`znchardecompose`(G,χ,Q)
complex Gauss sum $G_a(\chi)$	`znchargauss`(G,χ)

Conrey labelling

Conrey label $m\in(\mathbf{Z}/q\mathbf{Z})^*\to$ character	`znconreychar`(G,m)
character $\to$ Conrey label	`znconreyexp`(G,χ)
log on Conrey generators	`znconreylog`(G,m)
conductor of χ (χ_0 primitive)	`znconreyconductor`$(G,\chi,\{\chi_0\})$

True-False Tests

is x the disc. of a quadratic field?	`isfundamental`(x)
is x a prime?	`isprime`(x)
is x a strong pseudo-prime?	`ispseudoprime`(x)
is x square-free?	`issquarefree`(x)
is x a square?	`issquare`$(x,\{\&n\})$
is x a perfect power?	`ispower`$(x,\{k\},\{\&n\})$
is x a perfect power of a prime? ($x=p^n$)	`isprimepower`$(x,\&n)$
... of a pseudoprime?	`ispseudoprimepower`$(x,\&n)$
is x powerful?	`ispowerful`(x)
is x a totient? ($x=\varphi(n)$)	`istotient`$(x,\{\&n\})$
is x a polygonal number? ($x=P(s,n)$)	`ispolygonal`$(x,s,\{\&n\})$
is *pol* irreducible?	`polisirreducible`(pol)

Graphic Functions

crude graph of *expr* between a and b	`plot`$(X=a,b,expr)$

High-resolution plot (immediate plot)

plot *expr* between a and b	`ploth`$(X=a,b,expr,\{flag\},\{n\})$
plot points given by lists lx, ly	`plothraw`$(lx,ly,\{flag\})$
terminal dimensions	`plothsizes()`

Rectwindow functions

init window w, with size x,y	`plotinit`(w,x,y)
erase window w	`plotkill`(w)
copy w to w_2 with offset (dx,dy)	`plotcopy`(w,w_2,dx,dy)
clips contents of w	`plotclip`(w)
scale coordinates in w	`plotscale`(w,x_1,x_2,y_1,y_2)
`ploth` in w	`plotrecth`$(w,X=a,b,expr,\{flag\},\{n\})$
`plothraw` in w	`plotrecthraw`$(w,data,\{flag\})$
draw window w_1 at (x_1,y_1), ...	`plotdraw`$([[w_1,x_1,y_1],\ldots])$

Low-level Rectwindow Functions

set current drawing color in w to c	`plotcolor`(w,c)
current position of cursor in w	`plotcursor`(w)
write s at cursor's position	`plotstring`(w,s)
move cursor to (x,y)	`plotmove`(w,x,y)
move cursor to $(x+dx,y+dy)$	`plotrmove`(w,dx,dy)
draw a box to (x_2,y_2)	`plotbox`(w,x_2,y_2)
draw a box to $(x+dx,y+dy)$	`plotrbox`(w,dx,dy)
draw polygon	`plotlines`$(w,lx,ly,\{flag\})$
draw points	`plotpoints`(w,lx,ly)
draw line to $(x+dx,y+dy)$	`plotrline`(w,dx,dy)
draw point $(x+dx,y+dy)$	`plotrpoint`(w,dx,dy)

Convert to Postscript or Scalable Vector Graphics

The format f is either `"ps"` or `"svg"`.

as `ploth`	`plothexport`$(f,X=a,b,expr,\{flag\},\{n\})$
as `plothraw`	`plothrawexport`$(f,lx,ly,\{flag\})$
as `plotdraw`	`plotexport`$(f,[[w_1,x_1,y_1],\ldots])$

Send comments and corrections to ⟨Karim.Belabas@math.u-bordeaux.fr⟩