`Circe1` (internal Version 2.2): Beam Spectra for Simulating Linear Collider Physics*

Thorsten Ohl†

University of Würzburg
Emil-Hilb-Weg 22
D-97089 Würzburg
Germany

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Abstract

I describe parameterizations of realistic $e^{\pm}$- and γ-beam spectra at future linear e^+e^--colliders. Emphasis is put on simplicity and reproducibility of the parameterizations, supporting reproducible physics simulations. The parameterizations are implemented in a library of distribution functions and event generators.

Contents

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†e-mail: `ohl@physik.uni-wuerzburg.de`

Program Summary:

- **Title of program:** `Circe1` (March 2014)
- **Program obtainable** by anonymous `ftp` from the host `crunch.ikp.physik.th-darmstadt.de` in the directory `pub/ohl/circe`.
- **Licensing provisions:** Free software under the GNU General Public License.
- **Programming language used:** Fortran77 originally, transferred to Fortran90
- **Number of program lines in distributed program, including test data, etc.:** $\approx$ 1100 (excluding comments)
- **Computer/Operating System:** Any with a Fortran90 programming environment.
- **Memory required to execute with typical data:** Negligible on the scale of typical applications calling the library.
- **Typical running time:** A small fraction (typically a few percent) of the running time of applications calling the library.
- **Purpose of program:** Provide simple and reproducible, yet realistic, parameterizations of the $e^{\pm}$- and γ-beam spectra for linear colliders.
- **Nature of physical problem:** The intricate beam dynamics in the interaction region of a high luminosity linear collider at $\sqrt{s} = 500\text{GeV}$ result in non-trivial energy spectra of the scattering electrons, positrons and photons. Physics simulations require simple and reproducible, yet realistic, parameterizations of these spectra.
- **Method of solution:** Parameterization, curve fitting, Monte Carlo event generation.
- **Keywords:** Event generation, beamstrahlung, linear colliders.

1 Introduction

Despite the enormous quantitative success of the electro-weak standard model up to energies of 200GeV, neither the nature of electro-weak symmetry breaking (EWSB) nor the origin of mass are understood.

From theoretical considerations, we know that clues to the answer of these open questions are hidden in the energy range below $\Lambda_{\text{EWSB}} = 4\pi v \approx 3.1\text{TeV}$. Either we will discover a Higgs particle in this energy range or signatures for a strongly interacting EWSB sector will be found. Experiments at CERN's Large Hadron Collider (LHC) will shed a first light on this regime in the next decade. In the past is has been very fruitful to complement experiments at high energy hadron colliders with experiments at e^+e^--colliders. The simpler initial state allows more precise measurements with smaller theoretical errors. Lucid expositions of the physics opportunities of high energy e^+e^- colliders with references to the literature can be found in [1].

However, the power emitted by circular storage rings in form of synchrotron radiation scales like $(E/m)^4/R^2$ with the energy and mass of the particle and the radius of the ring. This cost becomes prohibitive after LEP2 and a Linear Collider (LC) has to be built instead.

Unfortunately, the "interesting" hard cross sections scale like $1/s$ with the square of the center of mass energy and a LC will have to operate at extremely high luminosities in excess of $10^{33}\text{cm}^{-2}\text{s}^{-1}$. To achieve such luminosities, the bunches of electrons and positrons have to be very dense. Under these conditions, the electrons undergo acceleration from strong electromagnetic forces from the positron bunch (and vice versa). The resulting synchrotron radiation is called *beamstrahlung* [2] and has a strong effect on the energy spectrum $D(x_1, x_2)$ of the colliding particles. This changes the observable e^+e^- cross sections

$$\frac{d\sigma_0^{e^+e^-}}{d\Omega}(s) \to \frac{d\sigma^{e^+e^-}}{d\Omega}(s) = \int_0^1 dx_1 dx_2\, D_{e^+e^-}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{e^+e^-}}{d\Omega'}(x_1 x_2 s) \tag{1a}$$

and produces luminosity for $e^\pm\gamma$ and $\gamma\gamma$ collisions:

$$\frac{d\sigma^{e^\pm\gamma}}{d\Omega}(s) = \int_0^1 dx_1 dx_2\, D_{e^\pm\gamma}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{e^\pm\gamma}}{d\Omega'}(x_1 x_2 s) \tag{1b}$$

$$\frac{d\sigma^{\gamma\gamma}}{d\Omega}(s) = \int_0^1 dx_1 dx_2\, D_{\gamma\gamma}(x_1, x_2; \sqrt{s}) J(\Omega', \Omega) \frac{d\sigma_0^{\gamma\gamma}}{d\Omega'}(x_1 x_2 s) \tag{1c}$$

Therefore, simulations of the physics expected at a LC need to know the spectra of the $e^\pm$ and γ beams precisely.

Microscopic simulations of the beam dynamics are available (e.g. `ABEL`[3], `CAIN`[4] and `Guinea-Pig`[5]) and their predictions are compatible with each other. But they require too much computer time and memory for direct use in physics programs. `Circe1` provides a fast and simple parameterization of the results from these simulations. Furthermore, even if the computational cost of the simulations would be negligible, the input parameters for microscopic simulations are not convenient for particle physics applications. Due to the highly

	SBAND	TESLA	XBAND	SBAND	TESLA	XBAND
E/GeV	250	250	250	500	500	500
$N_{\mathrm{particles}}/10^{10}$	1.1	3.63	0.65	2.9	1.8	0.95
$\epsilon_x/10^{-6}\mathrm{mrad}$	5	14	5	10	14	5
$\epsilon_y/10^{-6}\mathrm{mrad}$	0.25	0.25	0.08	0.1	0.06	0.1
β_x^*/mm	10.98	24.95	8.00	32	25	10.00
β_y^*/mm	0.45	0.70	0.13	0.8	0.7	0.12
σ_x/nm	335	845	286	571.87	598.08	226
σ_y/nm	15.1	18.9	4.52	9.04	6.55	3.57
$\sigma_z/\mu\mathrm{m}$	300	700	100	500	500	125
f_{rep}	50	5	180	50	5	180
n_{bunch}	333	1135	90	125	2270	90

Table 1: Accelerator parameters for three typical designs at $\sqrt{s} = 500\mathrm{GeV}$ and $\sqrt{s} = 1\mathrm{TeV}$. The resulting distributions are shown in figure 1. The design efforts are currently concentrated on a 350GeV-800GeV LC. Therefore the Tesla parameters for 1TeV are slightly out of date.

non-linear beam dynamics, the optimization of LC designs is a subtle art [6], that is best practiced by the experts. Furthermore, particle physics applications need benchmarking and easily reproducible parameterizations are required for this purpose.

The parameterizations in `Circe1` are not based on approximate solutions (cf. [7]) of the beamstrahlung dynamics. Instead, they provide a “phenomenological” description of the results from full simulations. The parameterizations are as simple as possible while remaining consistent with basic physical principles:

1. *positivity:* the distribution functions $D(x_1, x_2)$ *must not* be negative in the physical region $[0,1] \times [0,1]$.

2. *integrability:* the definite integral of the distribution functions over the physical region $[0,1] \times [0,1]$ *must* exist, even though the distributions can have singularities.

This paper is organized as follows: I start in section 2 with a discussion of the input for the microscopic simulations. In section 3 I describe the usage of the `Circe1` library and in section 4 I discuss some technical details of the implementation. After discussing the parameterizations available (in internal version 2.2) in section 5, I conclude in section 8.

2 Parameters

The microscopic simulation program `Guinea-Pig` [5] used for the current version of the parameterizations in `Circe1` simulates the passage of electrons through a

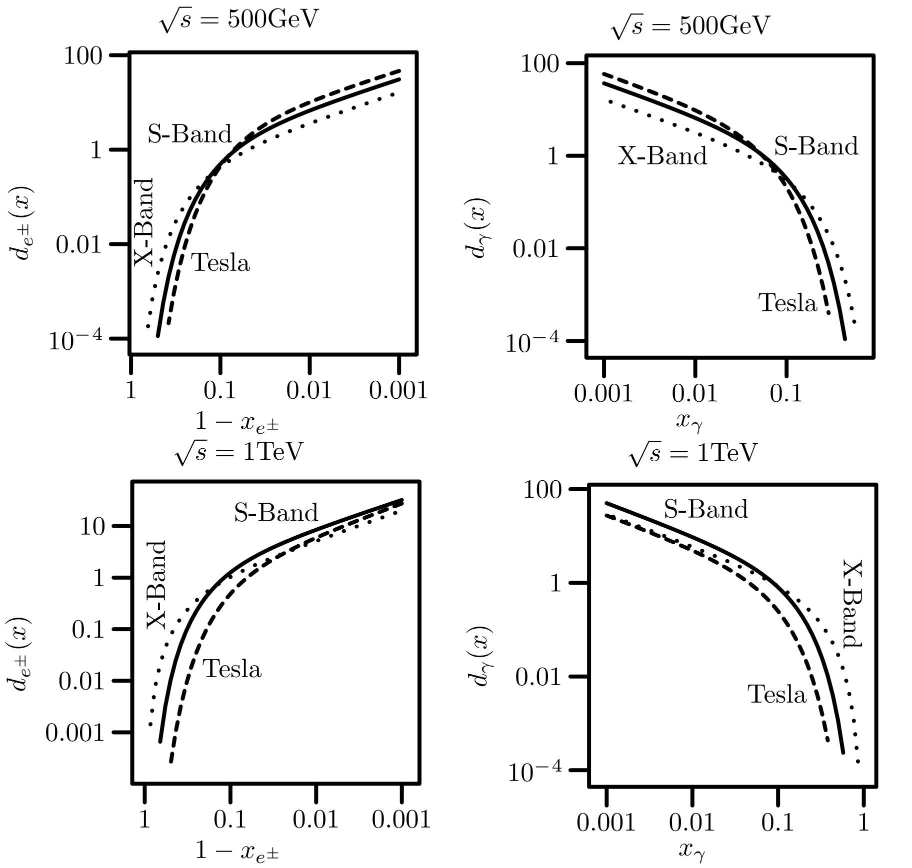

Figure 1: Version 1, revision 1996 09 02 of the factorized $e^{\pm}$- and γ-distributions at $\sqrt{s} = 500\text{GeV}$ and $\sqrt{s} = 1\text{TeV}$ in a doubly logarithmic plot. The accelerator parameters are taken from table 1.

	`TESLA`	`TESLA`	`TESLA`
E/GeV	175	250	400
$N_{\mathrm{particles}}/10^{10}$	3.63	3.63	3.63
$\epsilon_x/10^{-6}\mathrm{mrad}$	14	14	14
$\epsilon_y/10^{-6}\mathrm{mrad}$	0.25	0.25	0.1
β_x^*/mm	25.00	24.95	15.00
β_y^*/mm	0.70	0.70	0.70
σ_x/nm	1010.94	845	668.67
σ_y/nm	22.6	18.9	9.46
$\sigma_z/\mu\mathrm{m}$	700	700	700
f_{rep}	5	5	5
n_{bunch}	1135	1135	1135

Table 2: Accelerator parameters for the Tesla design at three planned [8] energies. The resulting distributions are shown in figure 2.

	High-$\mathcal{L}$	Low-$\mathcal{L}$	Low-ϵ_y
E/GeV	400	400	400
$N_{\mathrm{particles}}/10^{10}$	3.63	3.63	1.800
$\epsilon_x/10^{-6}\mathrm{mrad}$	14	14	12
$\epsilon_y/10^{-6}\mathrm{mrad}$	0.1	0.25	0.025
β_x^*/mm	15.00	25.00	25.00
β_y^*/mm	0.70	0.70	0.50
σ_x/nm	668.67	700.00	
σ_y/nm	9.46		
$\sigma_z/\mu\mathrm{m}$	700	700	500
f_{rep}	5	5	3
n_{bunch}	1135	1135	2260

Table 3: Variant accelerator parameters for the Tesla design at 800 Gev.

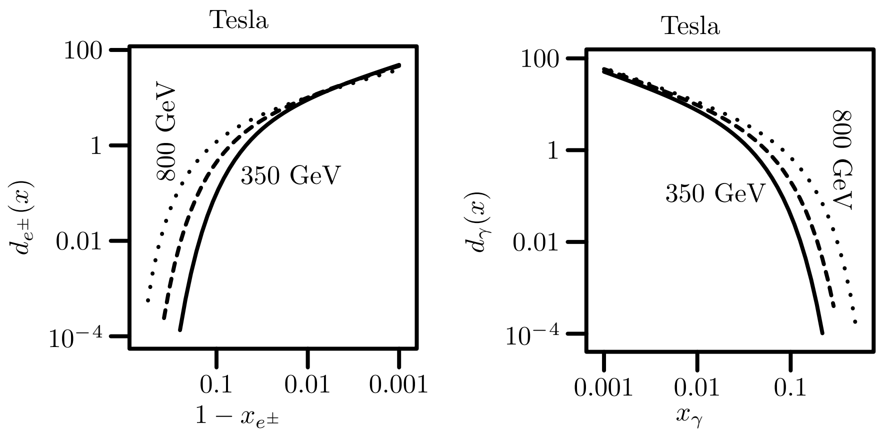

Figure 2: Version 1, revision 1996 09 02 of the factorized $e^\pm$- and γ-distributions for Tesla in a doubly logarithmic plot. The accelerator parameters are taken from table 2.

	TESLA	TESLA
E/GeV	250	400
$N_{\mathrm{particles}}/10^{10}$	2	1.40
$\epsilon_x/10^{-6}\mathrm{m\ rad}$	10	8
$\epsilon_y/10^{-6}\mathrm{m\ rad}$	0.03	0.01
β_x^*/mm	15.00	15.00
β_y^*/mm	0.40	0.30
σ_x/nm	553	391
σ_y/nm	5	2
$\sigma_z/\mu\mathrm{m}$	400	300
f_{rep}	5	3
n_{bunch}	2820	4500

Table 4: Accelerator parameters for a high luminosity Tesla design at two planned [8] energies. The resulting distributions are shown in figure 3.

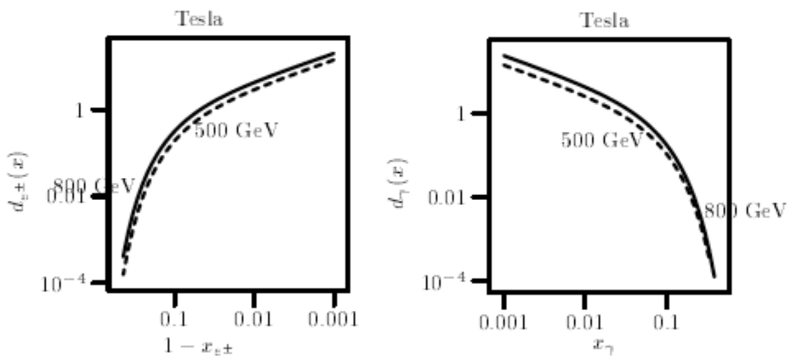

Figure 3: Version 5, revision 1998 05 05 of the factorized $e^\pm$- and γ-distributions for a high luminosity Tesla in a doubly logarithmic plot. The accelerator parameters are taken from table 4.

bunch of electrons (and vice versa). It takes the following accelerator parameters as input:

E : the energy of the particles before the beam-beam interaction.

$N_{\mathbf{particles}}$: the number of particles per bunch.

$\epsilon_{x,y}$: the normalized horizontal and vertical emittances.

$\beta^*_{x,y}$: the horizontal and vertical beta functions.

$\sigma_{x,y,z}$: the horizontal, vertical and longitudinal beam size. A Gaussian shape is used for the charge distribution in the bunches.

$f_{\mathbf{rep}}$: the repetition rate.

$n_{\mathbf{bunch}}$: the number of bunches per train.

The transversal beam sizes, beta functions and normalized emittances for relativistic particles are related by

$$\beta^*_{x,y} = \frac{\sigma^2_{x,y}}{\epsilon_{x,y}} \frac{E}{m_e} \tag{2}$$

The parameters used in the most recent revision of the parameterizations are collected in tables 1 and 2. The resulting factorized electron/positron and photon distributions in version 1 of the parameterizations are depicted in figures 1 and 2.

The most important purpose of `Circe1` is to map the manifold of possible beam spectra for the NLC to a *finite* number of *reproducible* parameterizations. The distributions

$$D^{\alpha\nu\rho}_{p_1 p_2}(x_1, x_2; \sqrt{s}) \tag{3}$$

provided by `Circe1` are indexed by three integers

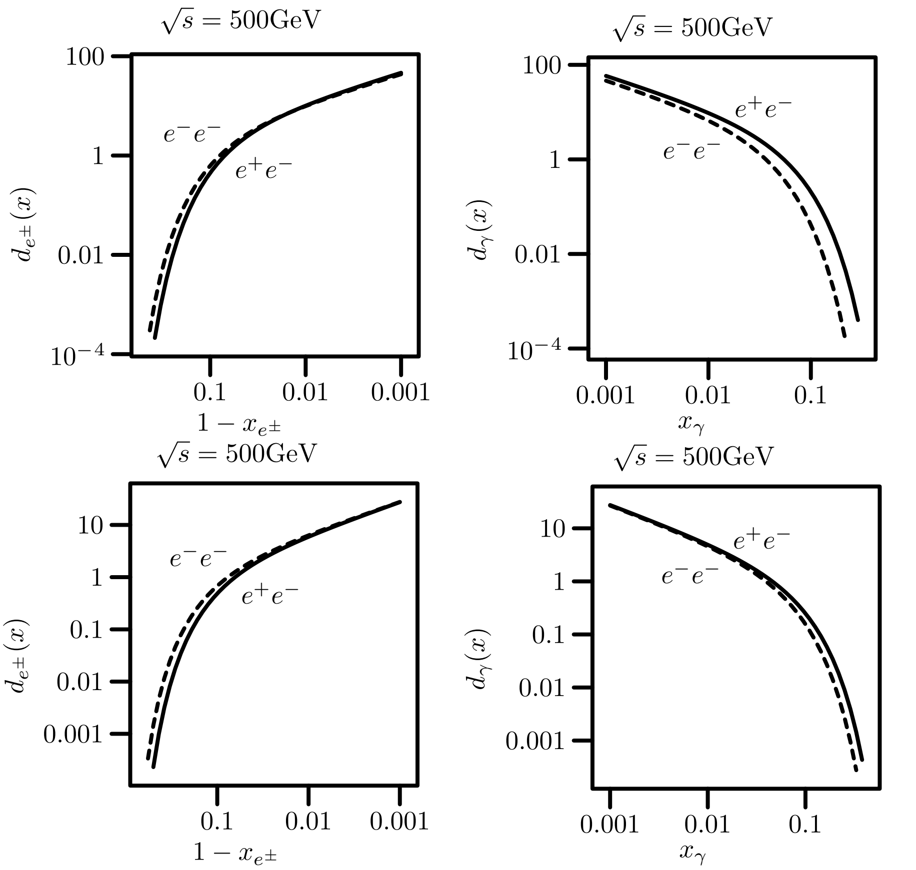

Figure 4: *Experimental:* Version 1, revision 0 of the factorized e^-- and γ-distributions for Tesla-e^-e^- in a doubly logarithmic plot. The accelerator parameters are taken from table 2 and have *not* been endorsed for use in an e^-e^--machine yet!.

α : the *accelerator design class:* currently there are three options: S-band [9], Tesla [8], X-band [10, 11]. More variety will be added later, in particular the e^-e^- mode and the $e^-\gamma$ and $\gamma\gamma$ laser backscattering modes of these designs.

ν : the *version of the parameterization:* over the years, the form of the parameterizations can change, either because better approximations are found or because new simulation programs become available. All versions will remain available in order to be able to reproduce calculations.

ρ : the *revision date for the parameterization:* a particular parameterization can contain bugs, which will be fixed in subsequent revisions. While only the most recent revision should be used for new calculations, old revisions will remain available in order to be able to reproduce calculations.

The continuous parameter $\sqrt{s}$ in (3) is misleading, because accelerator parameters have been optimized for discrete values of the energy. Therefore the distributions are not available for all values of $\sqrt{s}$.

The usage of the distributions in application programs is discussed in section 3.1. `Circe1` provides for each of the distributions a non-uniform random variate generator, that generates energy fractions according to the distributions. The usage of these generators is discussed in section 3.2.

3 Usage

3.1 Distributions

A generic interface to all distributions $D_{p_1p_2}(x_1, x_2)$ is given by the `circe` function

11a ⟨*API documentation* 11a⟩≡

```
function circe, d, x1, x2
  real(kind=double) :: circe
  integer :: p1, p2
  d = circe (x1, x2, p1, p2)
```

Uses `circe` 31b.

where the energy fractions are specified by $x_{1,2}$ and the particles $p_{1,2}$ are identified by their standard Monte Carlo codes (we use `C1` as a prefix to avoid name clashes when using `CIRCE1` inside `WHIZARD`):[13]

11b ⟨*Particle codes* 11b⟩≡

```
integer, parameter, public :: C1_ELECTRON = 11
integer, parameter, public :: C1_POSITRON = -11
integer, parameter, public :: C1_PHOTON = 22
```

Defines:
`C1_ELECTRON`, used in chunks 21e, 31b, 73b, 80c, and 81a.
`C1_PHOTON`, used in chunks 31b, 73b, 80c, 81a, and 87.
`C1_POSITRON`, used in chunks 22 and 81a.

The distributions can have integrable singularities at the end points, therefore the calling functions *must not* evaluate them at the endpoints 0 and 1. This is usually not a problem, since standard mapping techniques (cf. (10) below) will have to be used to take care of the singularity anyway. Nevertheless, all applications should favor open quadrature formulae (i.e. formulae not involving the endpoints) over closed formulae. The distributions are guaranteed to vanish unless $0 < x_{1,2} < 1$, with two exceptions. Firstly, the value -1 allows to pick up the integral of the continuum contribution:

$$D_{p_1p_2}(-1, x_2) = \lim_{\epsilon\to+0}\int_\epsilon^{1-\epsilon} dx_1\, D_{p_1p_2}(x_1, x_2) \tag{4a}$$

$$D_{p_1p_2}(x_1, -1) = \lim_{\epsilon\to+0}\int_\epsilon^{1-\epsilon} dx_2\, D_{p_1p_2}(x_1, x_2) \tag{4b}$$

$$D_{p_1p_2}(-1, -1) = \lim_{\epsilon\to+0}\int_\epsilon^{1-\epsilon} dx_1 dx_2\, D_{p_1p_2}(x_1, x_2) \tag{4c}$$

The other exception is that the strength of δ-function contributions at the endpoint can be picked up from the value at this endpoint:

$$D_{e^+e^-}(x_1, x_2) = D_{e^+e^-}(1,1)\delta(1-x_1)\delta(1-x_2) + \text{smooth and single }\delta \tag{5a}$$

$$D_{e^\pm\gamma}(x_1, x_2) = D_{e^\pm\gamma}(1,x_2)\delta(1-x_1) + \text{smooth} \tag{5b}$$

$$D_{\gamma e^\pm}(x_1, x_2) = D_{\gamma e^\pm}(x_1,1)\delta(1-x_2) + \text{smooth} \tag{5c}$$

The use of these special values is demonstrated in an example in section 3.1.1 below.

The distributions are normalized such that

$$\lim_{\epsilon\to+0}\int_{-\epsilon}^{1+\epsilon} dx_1 dx_2\, D_{e^+e^-}(x_1, x_2) = 1. \tag{6}$$

and the nominal e^+e^--luminosity of the currently active accelerator design can be retrieved from the database with the subroutine `circel`. The value is given in units of

$$\text{fb}^{-1}\upsilon^{-1} = 10^{32}\text{cm}^{-2}\text{sec}^{-1} \tag{7}$$

where $\upsilon = 10^7\text{sec} \approx \text{year}/\pi$ is an "effective year" of running with about 30% up-time.

12a ⟨*API documentation* 11a⟩+≡

```
real(kind=dobule) :: lumi
call circel (lumi)
```

Uses `circel` 41e.

A particular parameterization is selected by the `circes` function:

12b ⟨*API documentation* 11a⟩+≡

```
real(kind=double) :: x1m, x2m, roots
integer :: acc, ver, rev, chat
call circes (x1m, x2m, roots, acc, ver, rev, chat)
```

Uses `circes` 32a.

The parameter `roots` corresponds to the nominal center of mass energy $\sqrt{s}/\mathrm{GeV}$ of the collider. Currently $\sqrt{s} = 350\mathrm{GeV}, 500\mathrm{GeV}, 800\mathrm{GeV}, 1\mathrm{TeV}$ (i.e. `350D0`, `500D0`, `800D0` and `1000D0`) are supported. Application programs can *not* assume that energy values are interpolated. For convenience, e.g. in top threshold scans around 350GeV, a small interval around the supported values will be accepted as synonymous with the central value, but a warning will be printed. Section 5 should be consulted for the discrete values supported by a particular version of the parameterizations. Negative values of `roots` will keep the currently active value for $\sqrt{s}$.

The parameters `x1m` and `x2m` will set thresholds $x_{1,\min}$ and $x_{2,\min}$ for the event generation in the routines described in section 3.2.

The parameter `acc` selects the accelerator design. Currently the following accelerator codes are recognized:

13a ⟨*Accelerator codes* 13a⟩≡

```
integer, parameter :: SBAND  = 1
integer, parameter :: TESLA  = 2
integer, parameter :: XBAND  = 3
integer, parameter :: JLCNLC = 3
integer, parameter :: SBNDEE = 4
integer, parameter :: TESLEE = 5
integer, parameter :: XBNDEE = 6
integer, parameter :: NLCH   = 7
integer, parameter :: ILC    = 8
integer, parameter :: CLIC   = 9
```

Defines:
`CLIC`, used in chunk 35d.
`ILC`, used in chunks 35d, 69–71, and 92.
`JLCNLC`, used in chunks 17b, 18, 35d, 57a, 60–63, 66c, 67c, and 92.
`SBAND`, used in chunks 35d, 40, 44–47, 49b, 50a, 92, 107c, and 108c.
`SBNDEE`, used in chunks 34b, 35d, 41b, 44c, 46–49, and 92.
`TESLA`, used in chunks 32, 35d, 40, 44–47, 49–54, 56, 57, 59d, 60b, 62, 66b, 67b, 91b, 92, 107c, 108c, and 110a.
`TESLEE`, used in chunks 34b, 35d, 48, and 92.
`XBAND`, used in chunks 40, 44–47, 49b, 50a, 57d, 107c, and 108c.
`XBNDEE`, used in chunks 34b, 35d, 48, 49a, 91b, and 92.

The total number of accelerator codes

13b ⟨*Accelerator codes* 13a⟩+≡

```
integer, parameter :: NACC   = 9
```

Defines:
`NACC`, used in chunks 17b, 34, 35, 40, 41, 44–47, 51d, 53e, 56a, 59c, 62a, and 66a.

The `ver` parameter is used to determine the version as follows:

`ver` > 0 : a frozen version which is documented in section 5. For example, version 1 is a family of factorized Beta distributions: $D(x_1, x_2) \propto x_1^{a_1}(1-x_1)^{b_1} x_2^{a_2}(1-x_2)^{b_2}$.

`ver` $= 0$: the latest experimental version, which is usually not documented and can change at any time without announcement.

`ver` < 0 : keep the currently active version.

The `rev` parameter is used to determine the revision of a version as follows:

`rev` > 0 : a frozen revision which is documented in section 5. The integer `rev` is constructed from the date as follows: `rev` $= 10^4 \cdot \mathrm{year} + 10^2 \cdot \mathrm{month} + \mathrm{day}$, where the year is greater than 1995. Since Fortran77 ignored whitespace, it could be written like `1996 07 11` for readability. In Fortran90 the white space have been erased. If there is no exact match, the most recent revision before the specified date is chosen.

`rev` $= 0$: the most recent revision.

`rev` < 0 : keep the currently active revision.

Finally, the parameter `chat` controls the "chattiness" of `circe`. If it is 0, only error messages are printed. If it is 1, the parameters in use are printed whenever they change. Higher values of `chat` can produce even more diagnostics.

In addition to the generic interface `circe`, there are specialized functions for particular particle distributions. Obviously

$$D^{\alpha\nu\rho}_{e^\pm\gamma}(x_1, x_2, s) = D^{\alpha\nu\rho}_{\gamma e^\pm}(x_2, x_1, s) \tag{8}$$

and there are three independent functions $D_{e^-e^+}$, $D_{e^-\gamma}$ and $D_{\gamma\gamma}$ for the e^+e^- colliders with reasonable mnemonics:

14 ⟨*API documentation* 11a⟩+≡

```
real(kind=double) :: circee, circeg, circgg
d = circee (x1, x2)
d = circeg (x1, x2)
d = circgg (x1, x2)
```

Uses `circee` 41g, `circeg` 42c, and `circgg` 43c.

Calling the latter three functions is marginally faster in the current implementation, but this can change in the future.

3.1.1 Example

For clarification, let me give a simple example. Imagine we want to calculate the integrated production cross section

$$\sigma_X(s) = \int dx_1 dx_2\, \sigma_{e^+e^-\to X}(x_1 x_2 s) D_{e^+e^-}(x_1, x_2, s) \tag{9}$$

Since the distributions are singular in the $x_{1,2} \to 1$ limit, we have to map away this singularity with

$$x \to t = (1-x)^{1/\eta} \tag{10a}$$

Therefore

$$\int_0^1 dx\, f(x) = \int_0^1 dt\, \eta t^{\eta-1} f(1-t^\eta) \tag{10b}$$

with η sufficiently large to give the integrand a finite limit at $x \to 1$. If f diverges like a power $f(x) \propto 1/(1-x)^\beta$, this means $\eta > 1/(1-\beta)$.

As a specific example, let us "measure" a one particle s-channel exchange cross section

$$\sigma(s) \propto \frac{1}{s} \tag{11}$$

15a ⟨circe1_sample.f90: public 15a⟩≡
```
    public :: sigma
```
Uses sigma 15b.

15b ⟨circe1_sample.f90: subroutines 15b⟩≡
```
    function sigma (s)
      real(kind=double) :: s, sigma
      sigma = 1d0 / s
    end function sigma
```

Defines:
sigma, used in chunks 15–18 and 20d.

I will present the example code in a bottom-up fashion, which should be intuitive and is described in some more detail in appendix A. Assuming the existence of a one- and a two-dimensional Gaussian integration function gauss1 and gauss2,[1] we can perform the integral as follows:

15c ⟨*Gauss integration* 15c⟩≡
```
  s = sigma (1d0) * circee (1d0, 1d0) &
     + gauss1 (d1, 0d0, 1d0, EPS) &
     + gauss1 (d2, 0d0, 1d0, EPS) &
     + gauss2 (d12, 0d0, 1d0, 0d0, 1d0, EPS)
  write (*, 1000) 'delta(sigma) (Gauss) =', (s-1d0)*100d0
  1000 format (1X, A22, 1X, F6.2, '%')
```
Uses circee 41g, d1 16a, d12 15e, d2 16c, gauss1 89f, gauss2 90d, and sigma 15b.

Note how the four combinations of continuum and δ-peak are integrated separately, where you have to use three auxiliary functions d1, d2 and d12. The continuum contribution, including the Jacobian:

15d ⟨circe1_sample.f90: public 15a⟩+≡
```
    public :: d12
```
Uses d12 15e.

15e ⟨circe1_sample.f90: subroutines 15b⟩+≡
```
    function d12 (t1, t2)
      real(kind=double) :: d12, t1, t2, x1, x2
      ⟨EPS & PWR 16d⟩
      x1 = 1d0 - t1**PWR
      x2 = 1d0 - t2**PWR
      d12 = PWR*PWR * (t1*t2)**(PWR-1d0) &
              * sigma (x1*x2) * circee (x1, x2)
    end function d12
```

Defines:
d12, used in chunk 15.
Uses circee 41g and sigma 15b.

the first product of continuum and δ-peak:

15f ⟨circe1_sample.f90: public 15a⟩+≡
```
    public :: d1
```
Uses d1 16a.

[1]They are provided in the example program circe1_sample.f90.

16a ⟨circe1_sample.f90: subroutines 15b⟩+≡
```
function d1 (t1)
  real(kind=double) :: t1, x1, d1
  ⟨EPS & PWR 16d⟩
  x1 = 1d0 - t1**PWR
  d1 = PWR * t1**(PWR-1d0) * sigma (x1) * circee (x1, 1d0)
end function d1
```

Defines:
d1, used in chunks 15, 16, 41–43, and 74–77.
Uses circee 41g and sigma 15b.

and the second one:

16b ⟨circe1_sample.f90: public 15a⟩+≡
```
public :: d2
```
Uses d2 16c.

16c ⟨circe1_sample.f90: subroutines 15b⟩+≡
```
function d2 (t2)
  real(kind=double) :: t2, x2, d2
  ⟨EPS & PWR 16d⟩
  x2 = 1d0 - t2**PWR
  d2 = PWR * t2**(PWR-1d0) * sigma (x2) * circee (1d0, x2)
end function d2
```

Defines:
d2, used in chunks 15, 16, 41–43, and 74–77.
Uses circee 41g and sigma 15b.

Below you will see that the power of the singularity of the e^+e^- distributions at $x \to 1$ is $\approx -2/3$. To be on the safe side, we choose the power η in (10) as 5. It is kept in the parameter PWR, while EPS is the desired accuracy of the Gaussian integration:

16d ⟨EPS & PWR 16d⟩≡
```
real(kind=double), parameter :: EPS = 1d-6, PWR = 5d0
```

The Gauss integration of the non-singular version converges to the cotrrect value only if the final bin is integrated separately:

16e ⟨*Second Gauss integration* 16e⟩≡
```
s = gauss2 (d12a, 0d0, 1d0-KIREPS, 0d0, 1d0-KIREPS, EPS) &
  + gauss2 (d12a, 0d0, 1d0-KIREPS, 1d0-KIREPS, 1d0, EPS) &
  + gauss2 (d12a, 1d0-KIREPS, 1d0, 0d0, 1d0-KIREPS, EPS) &
  + gauss2 (d12a, 1d0-KIREPS, 1d0, 1d0-KIREPS, 1d0, EPS)
write (*, 1000) 'delta(sigma) (Gauss) =', (s-1d0)*100d0
```
Uses d12a 17a, gauss2 90d, and sigma 15b.

16f ⟨EPS & PWR 16d⟩+≡
```
real(kind=double), parameter :: KIREPS = 1D-6
```

16g ⟨circe1_sample.f90: public 15a⟩+≡
```
public :: d12a
```
Uses d12a 17a.

17a ⟨circe1_sample.f90: subroutines 15b⟩+≡

```
function d12a (x1, x2)
  real(kind=double) :: x1, x2, d12a
  d12a = sigma (x1*x2) * kirkee (x1, x2)
end function d12a
```

Defines:
  d12a, used in chunk 16.
Uses kirkee 74b and sigma 15b.

These code fragments can now be used in a main program that loops over energies and accelerator designs

17b ⟨circe1_sample.f90 17b⟩≡

```
! circe1_sample.f90 -- canonical beam spectra for linear collider physics
! $Id: prelude.nw 6466 2015-01-10 16:06:40Z jr_reuter $
⟨Copyleft notice 29b⟩
module sample_routines
  use kinds
  use circe1 !NODEP!

  implicit none
  private

⟨circe1_sample.f90: public 15a⟩

contains

⟨circe1_sample.f90: subroutines 15b⟩

end module sample_routines

  program circe1_sample
    use kinds
    use sample_routines
    use circe1

    implicit none

      ⟨Accelerator codes 13a⟩
      ⟨EPS & PWR 16d⟩
      ⟨Other variables in sample 19⟩
      integer :: acc, ver, i
      real(kind=double), dimension(9) :: roots(9) = &
         (/ 90D0,  170D0,  250D0,  350D0,  500D0, &
                  800D0, 1000D0, 1200D0, 1500D0 /)
      do acc = 1, NACC
      ! do acc = JLCNLC, NLCH, NLCH-JLCNLC
         do ver = 9, 9
            do i = 1, 9
               call circes (0d0, 0d0, roots(i), acc, ver, 20020328, 1)
               ⟨Gauss integration 15c⟩
```

```
                ⟨Second Gauss integration 16e⟩
                ⟨Monte Carlo integration 20d⟩
              end do
           end do
        end do
    end program circe1_sample
```

Uses JLCNLC 13a, NACC 13b, and circes 32a.

with the following result

18 ⟨*Sample output* 18⟩≡

```
circe1:message: starting up ...
circe1:message: $Id: prelude.nw 6466 2015-01-10 16:06:40Z jr_reuter $
circe1:message: updating 'roots' to   90.0
circe1:message: updating 'ver' to  7
circe1:message: updating 'rev' to 20000501
delta(sigma) (Gauss) =  0.11%
delta(sigma) (MC)    =  0.11%
                   +/-  0.00%
circe1:message: updating 'roots' to  170.0
circe1:message: updating 'ver' to  7
delta(sigma) (Gauss) =  0.38%
delta(sigma) (MC)    =  0.38%
                   +/-  0.01%
circe1:message: updating 'roots' to  350.0
circe1:message: updating 'ver' to  7
delta(sigma) (Gauss) =  1.67%
delta(sigma) (MC)    =  1.66%
                   +/-  0.03%
circe1:message: updating 'roots' to  500.0
circe1:message: updating 'ver' to  7
delta(sigma) (Gauss) =  3.66%
delta(sigma) (MC)    =  3.58%
                   +/-  0.07%
circe1:message: updating 'roots' to  800.0
circe1:message: updating 'ver' to  7
delta(sigma) (Gauss) =  5.21%
delta(sigma) (MC)    =  5.19%
                   +/-  0.11%
circe1:message: updating 'roots' to 1000.0
circe1:message: updating 'ver' to  7
circe1:message: energy 1000.0GeV too high, using spectrum for  800.0GeV
delta(sigma) (Gauss) =  5.21%
delta(sigma) (MC)    =  5.19%
                   +/-  0.11%
circe1:message: updating 'roots' to   90.0
circe1:message: updating 'acc' to JLCNLC
circe1:message: updating 'ver' to  7
circe1:message: energy   90.0GeV too low, using spectrum for  500.0GeV
delta(sigma) (Gauss) =  4.74%
```

```
delta(sigma) (MC)     =  4.75%
                    +/-  0.11%
circe1:message: updating 'roots' to  170.0
circe1:message: updating 'ver' to  7
circe1:message: energy  170.0GeV too low, using spectrum for  500.0GeV
delta(sigma) (Gauss) =  4.74%
delta(sigma) (MC)     =  4.68%
                    +/-  0.11%
circe1:message: updating 'roots' to  350.0
circe1:message: updating 'ver' to  7
circe1:message: energy  350.0GeV too low, using spectrum for  500.0GeV
delta(sigma) (Gauss) =  4.74%
delta(sigma) (MC)     =  4.75%
                    +/-  0.11%
circe1:message: updating 'roots' to  500.0
circe1:message: updating 'ver' to  7
delta(sigma) (Gauss) =  4.74%
delta(sigma) (MC)     =  4.75%
                    +/-  0.11%
circe1:message: updating 'roots' to  800.0
circe1:message: updating 'ver' to  7
circe1:message: energy  800.0GeV interpolated between  500.0 and 1000.0GeV
delta(sigma) (Gauss) =  8.37%
delta(sigma) (MC)     =  8.39%
                    +/-  0.21%
circe1:message: updating 'roots' to 1000.0
circe1:message: updating 'ver' to  7
delta(sigma) (Gauss) = 15.39%
delta(sigma) (MC)     = 14.68%
                    +/-  0.33%
```

Uses `JLCNLC` 13a and `sigma` 15b.

We almost forgot to declare the variables in the main program

19 ⟨*Other variables in* `sample` 19⟩≡

```
real(kind=double) :: s
```

This concludes the integration example. It should have made it obvious how to proceed in a realistic application.

In section 3.2.1 below, I will describe a Monte Carlo method for calculating such integrals efficiently.

3.2 Generators

The function `circe` and its companions are opaque to the user. Since they will in general contain singularities, applications will *not* be able to generate corresponding samples of random numbers efficiently. To fill this gap, four random number generators are provided. The subroutine `girce` will generate particle types $p_{1,2}$ and energy fractions $x_{1,2}$ in one step, according to the selected distribution.[2] Particle p_1 will be either a positron or a photon and p_2 will be

[2]The implementation of the flavor selection with non-vanishing thresholds $x_{1,\text{min}}$ and $x_{2,\text{min}}$ is moderately inefficient at the moment. It can be improved by a factor of two.

either an electron or a photon. The energy fractions are guaranteed to be above the currently active thresholds: $x_i \geq x_{i,\min}$. This can be used to cut on soft events—the photon distributions are rather soft—which might not be interesting in most simulations.

20a ⟨*API documentation* 11a⟩+≡
```
        call gircе  (x1, x2, p1, p2, rng)
```
Uses `girce` 80c.

The output parameters of `girce` are identical to the input parameters of `circe`, with the exception of `rng`. The latter is a subroutine with a single double precision argument, which will be assigned a uniform deviate from the interval $[0,1]$ after each call:

20b ⟨*API documentation* 11a⟩+≡
```
    subroutine rng (r)
      real(kind=double) :: r
      r = ⟨uniform deviate on [0, 1] (never defined)⟩
    end subroutine rng
```
Typically, it will be just a wrapper around the standard random number generator of the application program. For studies with a definite initial state, three generator functions are available.

20c ⟨*API documentation* 11a⟩+≡
```
        call gircee (x1, x2, rng)
        call girceg (x1, x2, rng)
        call gircgg (x1, x2, rng)
```
Uses `gircee` 81e, `girceg` 82c, and `gircgg` 83c.

3.2.1 Example

Returning to the example from section 3.2.1, I present a concise Monte Carlo algorithm for calculating the same integral:

20d ⟨*Monte Carlo integration* 20d⟩≡
```
  s = 0d0
  s2 = 0d0
  do n = 1, NEVENT
    call gircee (x1, x2, random)
    w = sigma (x1*x2)
    s = s + w
    s2 = s2 + w*w
  end do
  s = s / dble(NEVENT)
  s2 = s2 / dble(NEVENT)
  write (*, 1000) 'delta(sigma) (MC)    =', (s-1d0)*100d0
  write (*, 1000) '                   +/-', sqrt((s2-s*s)/dble(NEVENT))*100d0
```
Uses `gircee` 81e, `random` 21b, and `sigma` 15b.

20e ⟨*Other variables in* `sample` 19⟩+≡
```
      real(kind=double) :: w, s2, x1, x2
      integer, parameter :: NEVENT = 10000
      integer :: n
```

Here is a simple linear congruential random number generator for the sample program. Real applications will use their more sophisticated generators instead.

21a ⟨circe1_sample.f90: public 15a⟩+≡
```
    public :: random
```
Uses random 21b.

21b ⟨circe1_sample.f90: subroutines 15b⟩+≡
```
    subroutine random (r)
      real(kind=double), intent(out) :: r
      integer :: m = 259200, a = 7141, c = 54773
      integer, save :: n = 0
      ! data n /0/
      n = mod(n*a+c,m)
      r = real (n, kind=double) / real (m, kind=double)
    end subroutine random
```

Defines:
random, used in chunks 20 and 21.

If the cross section is slowly varying on the range where the $x_{1,2}$ distributions are non-zero, this algorithm is very efficient.

However, if this condition is not met, the explicit form of the parameterizations in section 5 should be consulted and appropriate mapping techniques should be applied. The typical example for this problem is a narrow resonance just below the nominal beam energy.

3.2.2 Event Generators

For Monte Carlo event generators that use the standard /hepevt/ common block [14], the addition of the Circe1 library is trivial. During the initialization of the event generator, the circes subroutine is called to set up Circe1's internal state. For example:

21c ⟨*Initialize event generator* 21c⟩≡
```
        call circes (0d0, 0d0, roots, acc, ver, 1996 07 11, 1)
```
Uses circes 32a.

During event generation, before setting up the e^+e^- initial state, the gircee subroutine is called with the event generator's random number generator:

21d ⟨*Event generation* 21d⟩≡
```
        call gircee (x1, x2, random)
```
Uses gircee 81e and random 21b.

The resulting energy fractions x_1 and x_2 are now available for defining the initial state electron

21e ⟨*Event generation* 21d⟩+≡
```
        isthep(1) = 101
        idhep(1) = C1_ELECTRON
        phep(1,1) = 0d0
        phep(2,1) = 0d0
        phep(3,1) = x1 * ebeam
        phep(4,1) = x1 * ebeam
        phep(5,1) = 0d0
```
Uses C1_ELECTRON 11b.

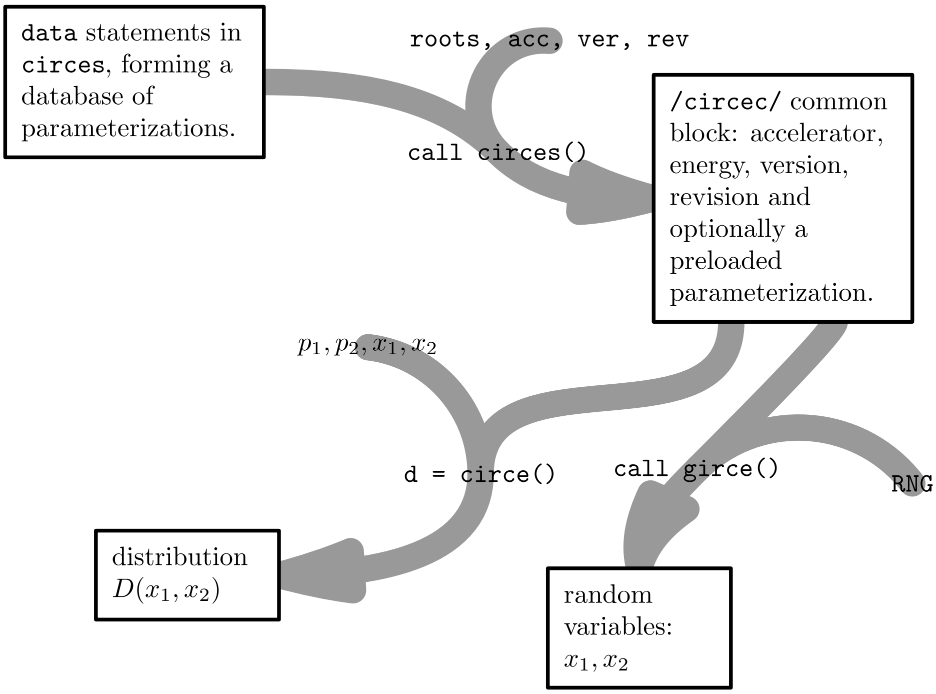

Figure 5: Architecture of `Circe1`: `circes()` selects energy and accelerator and loads the parameterization. The function `circe()` calculates the values of the selected distribution function at the given energy fractions. The subroutine `girce()` generates energy fractions using a specified random number generator in accordance with the selected distribution.

and positron.

22 ⟨*Event generation* 21d⟩+≡

```
        isthep(2) = 102
        idhep(2) = C1_POSITRON
        phep(1,2) = 0d0
        phep(2,2) = 0d0
        phep(3,2) = - x2 * ebeam
        phep(4,2) = x2 * ebeam
        phep(5,2) = 0d0
```

Uses `C1_POSITRON` 11b.

Using `Circe1` with other event generators should be straightforward as well.

4 Technical Notes

The structure of `Circe1` is extremely simple (cf. figure 5) and is mainly a bookkeeping excercise. All that needs to be done is to maintain a database of available parameterizations and to evaluate the corresponding functions. The only non trivial algorithms are used for the efficient generation of random deviates.

	SBAND	TESLA	TESLA'	XBAND
$\mathcal{L}/\mathrm{fb}^{-1}\upsilon^{-1}$	$31.38^{+0.22}_{-0.22}$	$106.25^{+0.71}_{-0.71}$	$95.24^{+0.73}_{-0.73}$	$36.39^{+0.29}_{-0.29}$
$\int d_{e^\pm}$	$0.4812^{+0.0041}_{-0.0041}$	$0.5723^{+0.0046}_{-0.0045}$	$0.3512^{+0.0048}_{-0.0048}$	$0.3487^{+0.0040}_{-0.0040}$
$x_{e^\pm}^\alpha$	$11.1534^{+0.0770}_{-0.0761}$	$15.2837^{+0.0923}_{-0.0914}$	$27.1032^{+0.3071}_{-0.3019}$	$6.9853^{+0.0733}_{-0.0718}$
$(1-x_{e^\pm})^\alpha$	$-0.6302^{+0.0013}_{-0.0012}$	$-0.6166^{+0.0011}_{-0.0011}$	$-0.6453^{+0.0017}_{-0.0017}$	$-0.6444^{+0.0017}_{-0.0017}$
$\int d_\gamma$	$0.6237^{+0.0033}_{-0.0033}$	$0.7381^{+0.0036}_{-0.0036}$	$0.3502^{+0.0034}_{-0.0034}$	$0.4149^{+0.0031}_{-0.0031}$
x_γ^α	$-0.6911^{+0.0006}_{-0.0006}$	$-0.6921^{+0.0006}_{-0.0006}$	$-0.6947^{+0.0011}_{-0.0011}$	$-0.6876^{+0.0010}_{-0.0010}$
$(1-x_\gamma)^\alpha$	$14.9355^{+0.0761}_{-0.0754}$	$24.1647^{+0.1124}_{-0.1116}$	$33.6576^{+0.3021}_{-0.2983}$	$8.3227^{+0.0659}_{-0.0649}$

Table 5: Version 1, revision 1997 04 16 of the beam spectra at 500 GeV. The rows correspond to the luminosity per effective year, the integral over the continuum and the powers in the factorized Beta distributions (12).

I have avoided the use of initialized `common` blocks (i.e. `block data` subroutines), because the Fortran77 standard does not provide a *portable* way of ensuring that `block data` subroutines are actually executed at loading time [3]. Instead, the `/circom/` common block is tagged by a "magic number" to check for initialization and its members are filled by the `circes` subroutine when necessary.

A more flexible method would be to replace the `data` statements by reading external files. This option causes portability problems, however, because I would have to make sure that the names of the external files are valid in all files systems of the target operating systems. More significantly, splitting the implementation into several parts forces the user to keep all files up to date. This can be a problem, because Fortran source files and data input files will typically be kept in different parts of the file system.

The option of implementing `Circe1` statelessly, i.e. with pure function calls and without `common` blocks, has been dismissed. While it would have been more straightforward on the side of the library, it would have placed the burdon of maintaining state (accelerator, energy, etc.) on the application program, thereby complicating them considerably. Keeping an explicit state in `Circe1` has the additional benefit of allowing to precompute certain internal variables, resulting in a more efficient implementation.

5 Parameterizations

The internal Version 2.2 of `Circe11` supports just one version of the parameterizations. Future versions will provide additional parameterizations.

5.1 Version 1

The first version of the parameterization uses a simple factorized *ansatz*

$$D_{p_1p_2}^{\alpha 1\rho}(x_1,x_2,s) = d_{p_1}^{\alpha 1\rho}(x_1)d_{p_2}^{\alpha 1\rho}(x_2) \tag{12a}$$

[3]In Fortran90 the common blocks have been replaced by saved module variables.

	SBAND	TESLA	TESLA'	XBAND
$\mathcal{L}/\mathrm{fb}^{-1}\upsilon^{-1}$	$119.00^{+0.83}_{-0.83}$	214.33^{+0***}_{-0***}	212.22^{+0***}_{-0***}	$118.99^{+0.91}_{-0.91}$
$\int d_{e^\pm}$	$0.5604^{+0.0040}_{-0.0039}$	$0.6686^{+0.0040}_{-0.0040}$	$0.4448^{+0.0043}_{-0.0043}$	$0.5001^{+0.0038}_{-0.0038}$
$x_{e^\pm}^\alpha$	$4.2170^{+0.0258}_{-0.0255}$	$5.5438^{+0.0241}_{-0.0239}$	$9.6341^{+0.0814}_{-0.0803}$	$2.6184^{+0.0192}_{-0.0190}$
$(1-x_{e^\pm})^\alpha$	$-0.6118^{+0.0013}_{-0.0013}$	$-0.5847^{+0.0011}_{-0.0011}$	$-0.6359^{+0.0014}_{-0.0014}$	$-0.6158^{+0.0015}_{-0.0015}$
$\int d_\gamma$	$0.7455^{+0.0032}_{-0.0032}$	$1.0112^{+0.0033}_{-0.0033}$	$0.4771^{+0.0031}_{-0.0031}$	$0.6741^{+0.0031}_{-0.0031}$
x_γ^α	$-0.6870^{+0.0006}_{-0.0006}$	$-0.6908^{+0.0004}_{-0.0004}$	$-0.6936^{+0.0008}_{-0.0008}$	$-0.6834^{+0.0007}_{-0.0007}$
$(1-x_\gamma)^\alpha$	$6.7145^{+0.0310}_{-0.0308}$	$9.9992^{+0.0342}_{-0.0340}$	$13.1607^{+0.0896}_{-0.0886}$	$3.8589^{+0.0215}_{-0.0213}$

Table 6: Version 1, revision 1997 04 17 of the beam spectra at 1 TeV.

	350 GeV	500 GeV	800 GeV	1600 GeV
$\mathcal{L}/\mathrm{fb}^{-1}\upsilon^{-1}$	$97.45^{+0.67}_{-0.67}$	$106.25^{+0.71}_{-0.71}$	170.86^{+0***}_{-0***}	340.86^{+0***}_{-0***}
$\int d_{e^\pm}$	$0.6093^{+0.0049}_{-0.0049}$	$0.5723^{+0.0046}_{-0.0045}$	$0.6398^{+0.0042}_{-0.0041}$	$0.5094^{+0.0040}_{-0.0040}$
$x_{e^\pm}^\alpha$	$17.6137^{+0.1065}_{-0.1055}$	$15.2837^{+0.0923}_{-0.0914}$	$7.6221^{+0.0365}_{-0.0361}$	$5.0550^{+0.0353}_{-0.0349}$
$(1-x_{e^\pm})^\alpha$	$-0.6061^{+0.0011}_{-0.0011}$	$-0.6166^{+0.0011}_{-0.0011}$	$-0.5944^{+0.0011}_{-0.0011}$	$-0.6187^{+0.0013}_{-0.0013}$
$\int d_\gamma$	$0.7729^{+0.0039}_{-0.0039}$	$0.7381^{+0.0036}_{-0.0036}$	$0.9178^{+0.0034}_{-0.0034}$	$0.5875^{+0.0031}_{-0.0031}$
x_γ^α	$-0.6949^{+0.0006}_{-0.0006}$	$-0.6921^{+0.0006}_{-0.0006}$	$-0.6908^{+0.0005}_{-0.0005}$	$-0.6892^{+0.0007}_{-0.0007}$
$(1-x_\gamma)^\alpha$	$28.9399^{+0.1370}_{-0.1361}$	$24.1647^{+0.1124}_{-0.1116}$	$13.1167^{+0.0497}_{-0.0495}$	$7.5514^{+0.0428}_{-0.0424}$

Table 7: Version 1, revision 1997 04 17 of the beam spectra for TESLA.

	500 GeV	800 GeV
$\mathcal{L}/\mathrm{fb}^{-1}\upsilon^{-1}$	$339.80^{+0.83}_{-0.83}$	$359.36^{+0.93}_{-0.93}$
$\int d_{e^\pm}$	$0.5019^{+0.0016}_{-0.0016}$	$0.4125^{+0.0016}_{-0.0016}$
$x_{e^\pm}^\alpha$	$12.2867^{+0.0318}_{-0.0316}$	$13.3242^{+0.0442}_{-0.0440}$
$(1-x_{e^\pm})^\alpha$	$-0.6276^{+0.0005}_{-0.0005}$	$-0.6401^{+0.0005}_{-0.0005}$
$\int d_\gamma$	$0.5114^{+0.0012}_{-0.0012}$	$0.3708^{+0.0011}_{-0.0011}$
x_γ^α	$-0.6912^{+0.0003}_{-0.0003}$	$-0.6924^{+0.0004}_{-0.0004}$
$(1-x_\gamma)^\alpha$	$17.0673^{+0.0375}_{-0.0375}$	$16.8145^{+0.0482}_{-0.0480}$

Table 8: Version 5, revision 1998 05 05 of the beam spectra for high luminosity TESLA.

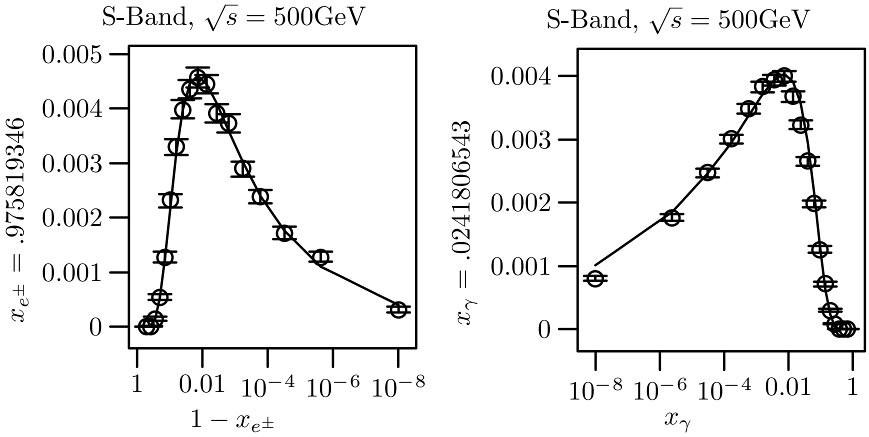

Figure 6: Fit of the $e^\pm$- and γ-distributions for the S-Band design at $\sqrt{s} = 500\text{GeV}$. The open circles with error bars are the result of the `Guinea-Pig` similation. The full line is the fit.

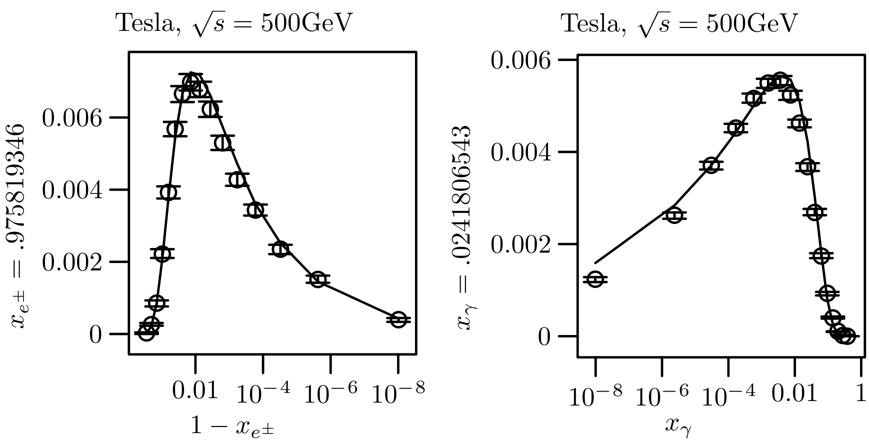

Figure 7: Fit of the $e^\pm$- and γ-distributions for the Tesla design at $\sqrt{s} = 500\text{GeV}$.

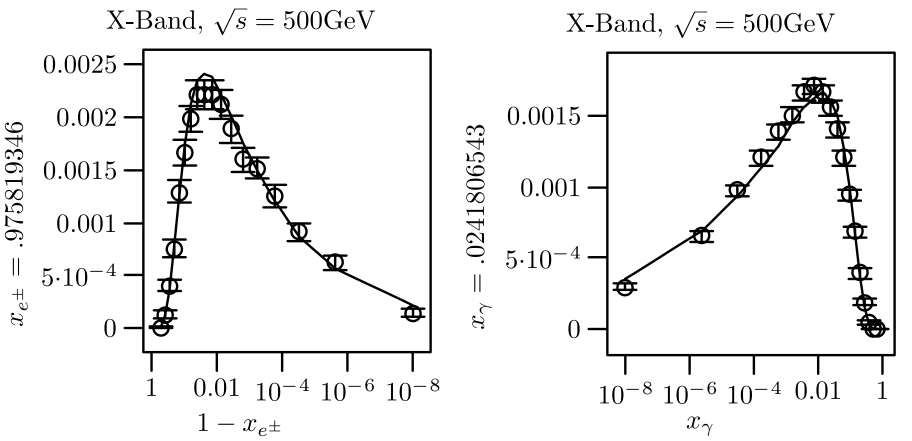

Figure 8: Fit of the $e^{\pm}$- and γ-distributions for the X-Band design at $\sqrt{s} =$ 500GeV.

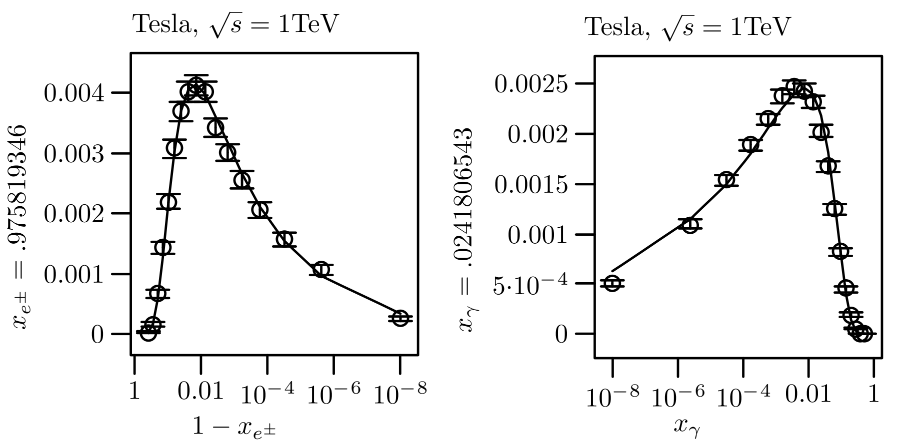

Figure 9: Fit of the $e^{\pm}$- and γ-distributions for the Tesla design at $\sqrt{s} = 1$TeV.

	SBNDEE	TESLEE	XBNDEE
$\mathcal{L}/\mathrm{fb}^{-1}\upsilon^{-1}$	$9.29^{+0.06}_{-0.06}$	$21.62^{+0.17}_{-0.17}$	$13.97^{+0.10}_{-0.10}$
$\int d_{e^\pm}$	$.6513^{+0.0059}_{-0.0059}$	$.7282^{+0.0083}_{-0.0082}$	$.5270^{+0.0049}_{-0.0049}$
$x^\alpha_{e^\pm}$	$10.3040^{+0.0601}_{-0.0593}$	$14.8578^{+0.1047}_{-0.1034}$	$5.8897^{+0.0455}_{-0.0448}$
$(1-x_{e^\pm})^\alpha$	$-.5946^{+0.0015}_{-0.0015}$	$-.5842^{+0.0018}_{-0.0018}$	$-.6169^{+0.0016}_{-0.0015}$
$\int d_\gamma$	$.4727^{+0.0035}_{-0.0035}$	$.5300^{+0.0046}_{-0.0046}$	$.3746^{+0.0029}_{-0.0029}$
x^α_γ	$-.6974^{+0.0009}_{-0.0009}$	$-.7039^{+0.0009}_{-0.0009}$	$-.6892^{+0.0010}_{-0.0010}$
$(1-x_\gamma)^\alpha$	$20.6447^{+0.1513}_{-0.1497}$	$36.1286^{+0.3027}_{-0.2991}$	$10.0872^{+0.0822}_{-0.0815}$

Table 9: *Experimental* Version 1, revision 0 of the beam spectra at 500 GeV. The rows correspond to the luminosity per effective year, the integral over the continuum and the powers in the factorized Beta distributions (12).

	SBNDEE	TESLEE	XBNDEE
$\mathcal{L}/\mathrm{fb}^{-1}\upsilon^{-1}$	$45.59^{+0.34}_{-0.34}$	$25.47^{+0.20}_{-0.20}$	$41.06^{+0.28}_{-0.28}$
$\int d_{e^\pm}$	$.7892^{+0.0075}_{-0.0074}$	$.6271^{+0.0066}_{-0.0065}$	$.7203^{+0.0058}_{-0.0058}$
$x^\alpha_{e^\pm}$	$5.4407^{+0.0285}_{-0.0281}$	$8.7504^{+0.0669}_{-0.0658}$	$2.7415^{+0.0121}_{-0.0119}$
$(1-x_{e^\pm})^\alpha$	$-.5285^{+0.0020}_{-0.0020}$	$-.6058^{+0.0017}_{-0.0017}$	$-.5049^{+0.0020}_{-0.0020}$
$\int d_\gamma$	$.6403^{+0.0040}_{-0.0040}$	$.4278^{+0.0038}_{-0.0038}$	$.6222^{+0.0032}_{-0.0032}$
x^α_γ	$-.6960^{+0.0008}_{-0.0008}$	$-.6982^{+0.0010}_{-0.0010}$	$-.6795^{+0.0008}_{-0.0008}$
$(1-x_\gamma)^\alpha$	$12.4803^{+0.0839}_{-0.0831}$	$18.5260^{+0.1674}_{-0.1655}$	$4.7506^{+0.0262}_{-0.0260}$

Table 10: *Experimental* Version 1, revision 0 of the beam spectra at 1 TeV.

where the distributions are simple Beta distributions:

$$d^{\alpha 1\rho}_{e^\pm}(x) = a^{\alpha\rho}_0 \delta(1-x) + a^{\alpha\rho}_1 x^{a^{\alpha\rho}_2} (1-x)^{a^{\alpha\rho}_3} \tag{12b}$$
$$d^{\alpha 1\rho}_{\gamma}(x) = a^{\alpha\rho}_4 x^{a^{\alpha\rho}_5} (1-x)^{a^{\alpha\rho}_6} \tag{12c}$$

This form of the distributions is motivated by the observation [2] that the $e^\pm$ distributions diverge like a power for $x \to 1$ and vanish at $x \to 0$. The behavior of the γ distributions is similar with the borders exchanged.

5.1.1 Fitting

The parameters a_i in (12) have been obtained by a least-square fit of (12) to histograms of simulation results from `Guinea-Pig`. Some care has to taken when fitting singular distributions to histogrammed data. Obviously equidistant bins are not a good idea, because most bins will be almost empty (cf. figures 1 and 2) and consequently a lot of information will be wasted. One solution to this problem is the use of logarithmic bins. This, however, maps the compact region $[0,1] \times [0,1]$ to $[-\infty,0] \times [-\infty,0]$, which is inconvenient because of the missing lower bounds.

	350 GeV	500 GeV	800 GeV
$\mathcal{L}/\mathrm{fb}^{-1}\upsilon^{-1}$	$15.18^{+0.13}_{-0.13}$	$21.62^{+0.17}_{-0.17}$	$43.98^{+0.38}_{-0.38}$
$\int d_{e^\pm}$	$.6691^{+0.0083}_{-0.0083}$	$.7282^{+0.0083}_{-0.0082}$	$.7701^{+0.0090}_{-0.0089}$
$x_{e^\pm}^\alpha$	$25.2753^{+0.2040}_{-0.2007}$	$14.8578^{+0.1047}_{-0.1034}$	$8.1905^{+0.0543}_{-0.0535}$
$(1-x_{e^\pm})^\alpha$	$-.5994^{+0.0017}_{-0.0017}$	$-.5842^{+0.0018}_{-0.0018}$	$-.5575^{+0.0021}_{-0.0021}$
$\int d_\gamma$	$.4464^{+0.0047}_{-0.0047}$	$.5300^{+0.0046}_{-0.0046}$	$.5839^{+0.0047}_{-0.0047}$
x_γ^α	$-.7040^{+0.0011}_{-0.0011}$	$-.7039^{+0.0009}_{-0.0009}$	$-.7046^{+0.0009}_{-0.0009}$
$(1-x_\gamma)^\alpha$	$60.1882^{+0.5882}_{-0.5797}$	$36.1286^{+0.3027}_{-0.2991}$	$19.3944^{+0.1681}_{-0.1660}$

Table 11: *Experimental* Version 1, revision 0 of the beam spectra for `TESLEE`.

The more appropriate solution is to use two maps

$$\begin{aligned} \phi : [0,1] &\to [0,1] \\ x &\mapsto y = x^{1/\eta} \end{aligned} \tag{13}$$

where $x = x_\gamma$ or $x = 1 - x_{e^\pm}$, and to bin the result equidistantly. If η is chosen properly (cf. (10)), the bin contents will then fall off at the singularity. The fits in tables 5, 6, and 7 have been performed with $\eta = 5$ and the resulting bin contents can be read off from figures 6–9.

Using this procedure for binning the results of the simulations, the popular fitting package `MINUIT` [15] converges quickly in all cases considered. The resulting parameters are given in tables 5, 6, and 7. Plots of the corresponding distributions have been shown in figures 1 and 2. It is obvious that an *ansatz* like (12) is able to distinguish among the accelerator designs. Thus it can provide a solid basis for physics studies.

In figures 6–9 I give a graphical impression of the quality of the fit, which appears to be as good as one could reasonably expect for a simple *ansatz* like (12). Note that the histograms have non-equidistant bins and that the resulting Jacobians have not been removed. Therefore the bin contents falls off at the singularities, as discussed above.

The errors used for the least-square fit had to be taken from a Monte Carlo (MC) study. `Guinea-Pig` only provides the $\sqrt{n}$ from Poissonian statistics for each bin, but the error accumulation during tracking the particles through phase space is not available. The MC studies shows that the latter error dominates the former, but appears to be reasonably Gaussian. A complete MC study of all parameter sets is computationally expensive (more than a week of processor time on a fast SGI). From an exemplary MC study of a few parameter sets, it appears that the errors can be described reasonably well by rescaling the Poissonian error in each bin with appropriate factors for electrons/positrons and photons and for continuum and delta. This procedure has been adopted.

The χ^2/d.o.f.'s of the fits are less than $\mathcal{O}(10)$. The simple *ansatz* (12) is therefore very satisfactory. In fact, trying to improve the ad-hoc factorized Beta distributions by the better motivated approximations from [7] or [16], it turns out [17] that (12) provides a significantly better fit of the results of the simulations. The price to pay is that the parameters in (12) have no direct physical interpretation.

5.1.2 Generators

For this version of the parameterizations we need a fast generator of Beta distributions:

$$\beta^{a,b}(x) \propto x^{a-1}(1-x)^{b-1} \tag{14}$$

This problem has been studied extensively and we can use a published algorithm [18] that is guaranteed to be very fast for all a, b such that $0 < a \leq 1 \leq b$, which turns out to be always the case (cf. tables 5, 6, and 7).

5.2 Future Versions

There are two ways in which the parameterizations can be improved:

more complicated functions: the factorized fits can only be improved marginally by adding more positive semi-definite factors to (12). More improvement is possible by using sums of functions, but in this case, the best fits violate the positivity requirement and have to be discarded.

correlations: the parameterization in section 5.1 is factorized. While this is a good approximation, the simulations nevertheless show correlations among x_1 and x_2. These correlations can be included in a future version.

interpolation: the parameterization in section 5.1 is based on fitting the simulation results by simple functions. Again, this appears to be a good approximation. But such fits can not uncover any fine structure of the distributions. Therefore it will be worthwhile to study interpolations of the simulation results in the future. A proper interpolation of results with statistical errors is however far from trivial: straightforward polynomial or spline interpolations will be oscillatory and violate the positivity requirement. Smoothing algorithms have to be investigated in depth before such a parameterization can be released.

other simulations: besides [5], other simulation codes are invited to contribute their results for inclusion in the `Circe1` library.

6 Implementation of `circe1`

29a ⟨`circe1.f90` 29a⟩≡

```
! circe1.f90 -- canonical beam spectra for linear collider physics
! $Id: circe1.nw 6466 2015-01-10 16:06:40Z jr_reuter $
```

⟨*Copyleft notice* 29b⟩
⟨*Main module* 30b⟩

29b ⟨*Copyleft notice* 29b⟩≡

```
!
! Copyright (C) 1999-2015 by
!     Wolfgang Kilian <kilian@physik.uni-siegen.de>
!     Thorsten Ohl <ohl@physik.uni-wuerzburg.de>
```

```
!     Juergen Reuter <juergen.reuter@desy.de>
!     with contributions from
!     Christian Speckner <cnspeckn@googlemail.com>
!
! WHIZARD is free software; you can redistribute it and/or modify it
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!
! You should have received a copy of the GNU General Public License
! along with this program; if not, write to the Free Software
! Foundation, Inc., 675 Mass Ave, Cambridge, MA 02139, USA.
!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
! This file has been stripped of most comments.  For documentation, refer
! to the source 'circe1.nw'
```

Now we can move on to the implementation.

6.1 Symbolic Constants

The file `circe.h` contains symbolic names for various magic constants used by `Circe1`:

30a ⟨`circe.h` 30a⟩≡

```
c circe.h -- canonical beam spectra for linear collider physics
c $Id: circe1.nw 6466 2015-01-10 16:06:40Z jr_reuter $
```

Uses `circe` 31b.

30b ⟨*Main module* 30b⟩≡

```
module circe1
  use kinds

  implicit none
  private
  ⟨Public subroutines 31a⟩

  ⟨Public types 79d⟩

  ⟨Particle codes 11b⟩
  ⟨Accelerator codes 13a⟩
  ⟨Private parameters 37c⟩

  integer, parameter, public :: MAGIC0 = 19040616
  real(kind=double), parameter :: KIREPS = 1D-6

  ⟨Declaration: circe1 parameters 32c⟩
```

```
  type(circe1_params_t), public, save :: circe1_params

  ⟨Abstract types 79e⟩

  ⟨Abstract interfaces 79c⟩

contains

  ⟨Module subroutines 31b⟩
end module circe1
```

6.2 Distributions

6.2.1 Version 1

We start with a convenience function which dispatches over the valid particle types. The hardest part is of course to avoid typos in such trivial functions ...

31a ⟨*Public subroutines* 31a⟩≡

```
public :: circe
```

Uses circe 31b.

31b ⟨*Module subroutines* 31b⟩≡

```
function circe (x1, x2, p1, p2)
  real(kind=double) :: x1, x2
  integer :: p1, p2
  real(kind=double) :: circe
  ⟨Initialization check 32g⟩
    circe = -1.0
    if (abs(p1) .eq. C1_ELECTRON) then
       if (abs(p2) .eq. C1_ELECTRON) then
          circe = circee (x1, x2)
       else if (p2 .eq. C1_PHOTON) then
          circe = circeg (x1, x2)
       end if
    else if (p1 .eq. C1_PHOTON) then
       if (abs(p2) .eq. C1_ELECTRON) then
          circe = circeg (x2, x1)
       else if (p2 .eq. C1_PHOTON) then
          circe = circgg (x1, x2)
       end if
    end if
end function circe
```

Defines:
circe, used in chunks 11a, 31, 30a, 31a, 87, 93b, 110b, and 115d.
Uses C1_ELECTRON 11b, C1_PHOTON 11b, circee 41g, circeg 42c, and circgg 43c.

31c ⟨*Public subroutines* 31a⟩+≡

```
public :: circes
```

Uses circes 32a.

32a ⟨*Module subroutines* 31b⟩+≡
```
  subroutine circes (xx1m, xx2m, xroots, xacc, xver, xrev, xchat)
    real(kind=double) :: xx1m, xx2m, xroots
    integer :: xacc, xver, xrev, xchat
⟨Local variables for circes 33b⟩
⟨Initializations for circes 35b⟩
    if (circe1_params%magic .ne. 19040616) then
       circe1_params%magic = 19040616
       ⟨Initialize circe1 parameters 32h⟩
    end if
⟨Update circe1 parameters 33a⟩
⟨formats for circes 38d⟩
  end subroutine circes
```
Defines:
  `circes`, used in chunks 32a, 12b, 17b, 32a, 21c, 31, 32, 35f, 87, and 91b.

32b ⟨*Public subroutines* 31a⟩+≡
```
  public :: circe1_params_t
```

32c ⟨*Declaration: circe1 parameters* 32c⟩≡
```
  type :: circe1_params_t
⟨8-byte aligned part of circe1 parameters 32d⟩
⟨4-byte aligned part of circe1 parameters 32e⟩
  end type circe1_params_t
```

32d ⟨*8-byte aligned part of circe1 parameters* 32d⟩≡
```
    real(kind=double) :: x1m = 0d0
    real(kind=double) :: x2m = 0d0
    real(kind=double) :: roots = 500D0
```

32e ⟨*4-byte aligned part of circe1 parameters* 32e⟩≡
```
    integer :: acc = TESLA
    integer :: ver = 0
    integer :: rev = 0
    integer :: chat = 1
```
Uses `TESLA` 13a.

Instead of using fragile `block data` subroutines, we use a magic number to tag `circe1_params` as initialized:

32f ⟨*4-byte aligned part of circe1 parameters* 32e⟩+≡
```
    integer :: magic
```

Since negative values are no updated, we can call `circes` with all negative variables to ensure initialization:

32g ⟨*Initialization check* 32g⟩≡
```
      if (circe1_params%magic .ne. MAGIC0) then
         call circes (-1d0, -1d0, -1d0, -1, -1, -1, -1)
      endif
```
Uses `circes` 32a.

32h ⟨*Initialize circe1 parameters* 32h⟩≡
```
      circe1_params%x1m = 0d0
      circe1_params%x2m = 0d0
```

```
circe1_params%roots = 500D0
circe1_params%acc = TESLA
circe1_params%ver = 0
circe1_params%rev = 0
circe1_params%chat = 1
if (xchat .ne. 0) then
   call circem ('MESSAGE', 'starting up ...')
   call circem ('MESSAGE', &
      '$Id: circe1.nw 6466 2015-01-10 16:06:40Z jr_reuter $')
endif
```

Uses TESLA 13a and circem 86e.

33a ⟨*Update circe1 parameters* 33a⟩≡

```
      if ((xchat .ge. 0) .and. (xchat .ne. circe1_params%chat)) then
         circe1_params%chat = xchat
         if (circe1_params%chat .ge. 1) then
            write (msgbuf, 1000) 'chat', circe1_params%chat
1000        format ('updating `', A, ''' to ', I2)
            call circem ('MESSAGE', msgbuf)
         endif
      else
         if (circe1_params%chat .ge. 2) then
            write (msgbuf, 1100) 'chat', circe1_params%chat
1100        format ('keeping `', A, ''' at ', I2)
            call circem ('MESSAGE', msgbuf)
         endif
      endif
```

Uses circem 86e.

33b ⟨*Local variables for* circes 33b⟩≡

```
character(len=60) :: msgbuf
```

33c ⟨*Update circe1 parameters* 33a⟩+≡

```
      if ((xx1m .ge. 0d0) .and. (xx1m .ne. circe1_params%x1m)) then
         circe1_params%x1m = xx1m
         if (circe1_params%chat .ge. 1) then
            write (msgbuf, 1001) 'x1min', circe1_params%x1m
1001        format ('updating `', A, ''' to ', E12.4)
            call circem ('MESSAGE', msgbuf)
         endif
      else
         if (circe1_params%chat .ge. 2) then
            write (msgbuf, 1101) 'x1min', circe1_params%x1m
1101        format ('keeping `', A, ''' at ', E12.4)
            call circem ('MESSAGE', msgbuf)
         endif
      endif
```

Uses circem 86e.

33d ⟨*Update circe1 parameters* 33a⟩+≡

```
      if ((xx2m .ge. 0d0) .and. (xx2m .ne. circe1_params%x2m)) then
         circe1_params%x2m = xx2m
```

```
        if (circe1_params%chat .ge. 1) then
           write (msgbuf, 1001) 'x2min', circe1_params%x2m
           call circem ('MESSAGE', msgbuf)
        endif
     else
        if (circe1_params%chat .ge. 2) then
           write (msgbuf, 1101) 'x2min', circe1_params%x2m
           call circem ('MESSAGE', msgbuf)
        endif
     endif
```
Uses `circem` 86e.

34a ⟨*Update circe1 parameters* 33a⟩+≡
```
     if ((xroots .ge. 0d0) .and.(xroots .ne. circe1_params%roots)) then
        circe1_params%roots = xroots
        if (circe1_params%chat .ge. 1) then
           write (msgbuf, 1002) 'roots', circe1_params%roots
1002       format ('updating `', A, ''' to ', F6.1)
           call circem ('MESSAGE', msgbuf)
        endif
     else
        if (circe1_params%chat .ge. 2) then
           write (msgbuf, 1102) 'roots', circe1_params%roots
1102       format ('keeping `', A, ''' at ', F6.1)
           call circem ('MESSAGE', msgbuf)
        endif
     endif
```
Uses `circem` 86e.

34b ⟨*Update circe1 parameters* 33a⟩+≡
```
     if ((xacc .ge. 0) .and.(xacc .ne. circe1_params%acc)) then
        if ((xacc .ge. 1) .and. (xacc .le. NACC)) then
           circe1_params%acc = xacc
           if (circe1_params%chat .ge. 1) then
              write (msgbuf, 1003) 'acc', accnam(circe1_params%acc)
1003          format ('updating `', A, ''' to ', A)
              call circem ('MESSAGE', msgbuf)
           endif
        else
           write (msgbuf, 1203) xacc
1203       format ('invalid `acc'': ', I8)
           call circem ('ERROR', msgbuf)
           write (msgbuf, 1103) 'acc', accnam(circe1_params%acc)
1103       format ('keeping `', A, ''' at ', A)
           call circem ('MESSAGE', msgbuf)
        endif
     else
        if (circe1_params%chat .ge. 2) then
           write (msgbuf, 1003) 'acc', accnam(circe1_params%acc)
           call circem ('MESSAGE', msgbuf)
        endif
```

```
        endif
        if ((circe1_params%acc .eq. SBNDEE) .or. (circe1_params%acc .eq. TESLEE) &
           .or. (circe1_params%acc .eq. XBNDEE)) then
```
⟨*Warn that no parameter set has been endorsed for e^-e^- yet* 36a⟩
```
        endif
```
Uses `NACC` 13b, `SBNDEE` 13a, `TESLEE` 13a, `XBNDEE` 13a, and `circem` 86e.

35a ⟨*Local variables for* `circes` 33b⟩+≡
⟨*Declaration of* `accnam` 35c⟩

35b ⟨*Initializations for* `circes` 35b⟩≡
⟨*Initialization of* `accnam` 35d⟩

35c ⟨*Declaration of* `accnam` 35c⟩≡
```
        character(len=6), dimension(NACC) :: accnam
```
Uses `NACC` 13b.

35d ⟨*Initialization of* `accnam` 35d⟩≡
```
        data accnam(SBAND)  /'SBAND'/
        data accnam(TESLA)  /'TESLA'/
        data accnam(JLCNLC) /'JLCNLC'/
        data accnam(SBNDEE) /'SBNDEE'/
        data accnam(TESLEE) /'TESLEE'/
        data accnam(XBNDEE) /'XBNDEE'/
        data accnam(NLCH) /'NLC H'/
        data accnam(ILC) /'ILC'/
        data accnam(CLIC) /'CLIC'/
```
Uses `CLIC` 13a, `ILC` 13a, `JLCNLC` 13a, `SBAND` 13a, `SBNDEE` 13a, `TESLA` 13a, `TESLEE` 13a, and `XBNDEE` 13a.

35e ⟨*Public subroutines* 31a⟩+≡
```
    public :: circex
```
Uses `circex` 35f.

35f ⟨*Module subroutines* 31b⟩+≡
```
    subroutine circex (xx1m, xx2m, xroots, cacc, xver, xrev, xchat)
      real(kind=double) :: xx1m, xx2m, xroots
      character(*) :: cacc
      integer :: xver, xrev, xchat
      integer :: xacc, i
```
⟨*Accelerator codes* 13a⟩
⟨*Declaration of* `accnam` 35c⟩
⟨*Initialization of* `accnam` 35d⟩
```
      xacc = -1
      do i = 1, NACC
         if (trim (accnam(i)) == trim (cacc)) then
           xacc = i
         end if
      end do
      call circes (xx1m, xx2m, xroots, xacc, xver, xrev, xchat)
    end subroutine circex
```
Defines:
`circex`, used in chunk 35e.
Uses `NACC` 13b and `circes` 32a.

36a ⟨*Warn that no parameter set has been endorsed for e^-e^- yet* 36a⟩≡
```
        call circem ('WARNING', '*************************************')
        call circem ('WARNING', '* The accelerator parameters have *')
        call circem ('WARNING', '* not been endorsed for use in    *')
        call circem ('WARNING', '* an e-e- collider yet!!!         *')
        call circem ('WARNING', '*************************************')
```
Uses `circem` 86e.

36b ⟨*Update circe1 parameters* 33a⟩+≡
```
        if (xver .ge. 0) then
           circe1_params%ver = xver
           if (circe1_params%chat .ge. 1) then
              write (msgbuf, 1000) 'ver', circe1_params%ver
              call circem ('MESSAGE', msgbuf)
           endif
        else
           if (circe1_params%chat .ge. 2) then
              write (msgbuf, 1100) 'ver', circe1_params%ver
              call circem ('MESSAGE', msgbuf)
           endif
        endif
```
Uses `circem` 86e.

36c ⟨*Update circe1 parameters* 33a⟩+≡
```
        if ((xrev .ge. 0) .and.(xrev .ne. circe1_params%rev)) then
           circe1_params%rev = xrev
           if (circe1_params%chat .ge. 1) then
              write (msgbuf, 1004) 'rev', circe1_params%rev
 1004         format ('updating `', A, ''' to ', I8)
              call circem ('MESSAGE', msgbuf)
           endif
        else
           if (circe1_params%chat .ge. 2) then
              write (msgbuf, 1104) 'rev', circe1_params%rev
 1104         format ('keeping `', A, ''' at ', I8)
              call circem ('MESSAGE', msgbuf)
           endif
        endif
```
Uses `circem` 86e.

Versions 3 and 4 are identical to version 1, except for TESLA at 800 GeV.

36d ⟨*Update circe1 parameters* 33a⟩+≡
```
        ver34 = 0
        if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
    ⟨Update version 1 derived parameters in circe1 parameters 37d⟩
        else if ((circe1_params%ver .eq. 3) .or. (circe1_params%ver .eq. 4)) then
           ver34 = circe1_params%ver
           circe1_params%ver = 1
    ⟨Update version 3 and 4 derived parameters in circe1 parameters 50c⟩
        else if (circe1_params%ver .eq. 5) then
           circe1_params%ver = 1
```

```
⟨Update version 5 derived parameters in circe1 parameters 53a⟩
      else if (circe1_params%ver .eq. 6) then
         circe1_params%ver = 1
⟨Update version 6 derived parameters in circe1 parameters 54c⟩
      else if (circe1_params%ver .eq. 7) then
         circe1_params%ver = 1
⟨Update version 7 derived parameters in circe1 parameters 56d⟩
      else if (circe1_params%ver .eq. 8) then
         circe1_params%ver = 1
⟨Update version 8 derived parameters in circe1 parameters 61a⟩
      else if (circe1_params%ver .eq. 9) then
         circe1_params%ver = 1
⟨Update version 9 derived parameters in circe1 parameters 63a⟩
      else if (circe1_params%ver .eq. 10) then
         circe1_params%ver = 1
⟨Update version 10 derived parameters in circe1 parameters 68b⟩
⟨else handle invalid versions 37b⟩
```

37a ⟨*Local variables for* `circes` 33b⟩+≡

```
      integer :: ver34
```

37b ⟨`else` *handle invalid versions* 37b⟩≡

```
      else if (circe1_params%ver .eq. 2) then
⟨Version 2 has been retired 50b⟩
      else if (circe1_params%ver .gt. 10) then
         call circem ('PANIC', 'versions >10 not available yet')
         return
      else
         call circem ('PANIC', 'version must be positive')
         return
      end if
```

Uses `circem` 86e.

37c ⟨*Private parameters* 37c⟩≡

```
   integer :: e, r, ehi, elo
```

37d ⟨*Update version 1 derived parameters in circe1 parameters* 37d⟩≡

```
      if (circe1_params%rev .eq. 0) then
         r = 0
      elseif (circe1_params%rev .ge. 19970417) then
         r = 5
      elseif (circe1_params%rev .ge. 19960902) then
         r = 4
      elseif (circe1_params%rev .ge. 19960729) then
         r = 3
      elseif (circe1_params%rev .ge. 19960711) then
         r = 2
      elseif (circe1_params%rev .ge. 19960401) then
         r = 1
      elseif (circe1_params%rev .lt. 19960401) then
         call circem ('ERROR', &
             'no revision of version 1 before 96/04/01 available')
```

```
      call circem ('MESSAGE', 'falling back to default')
      r = 1
   endif
   if (circe1_params%chat .ge. 2) then
      write (msgbuf, 2000) circe1_params%rev, r
2000  format ('mapping date ', I8, ' to revision index ', I2)
      call circem ('MESSAGE', msgbuf)
   endif
```

Uses circem 86e.

38a ⟨*Log revision mapping* 38a⟩≡

```
   if (circe1_params%chat .ge. 2) then
      write (msgbuf, 2000) circe1_params%rev, r
      call circem ('MESSAGE', msgbuf)
   endif
```

Uses circem 86e.

38b ⟨*Update version 1 derived parameters in circe1 parameters* 37d⟩+≡

⟨*Map* roots *to* e 38c⟩

38c ⟨*Map* roots *to* e 38c⟩≡

```
   if (circe1_params%roots .eq. 350d0) then
      e = GEV350
   else if ((circe1_params%roots .ge. 340d0) .and. (circe1_params%roots .le. 370d0))
      write (msgbuf, 2001) circe1_params%roots, 350d0
      call circem ('MESSAGE', msgbuf)
      e = GEV350
```

Uses circem 86e.

38d ⟨formats *for* circes 38d⟩≡

```
2001 format ('treating energy ', F6.1, 'GeV as ',  F6.1, 'GeV')
```

38e ⟨*Map* roots *to* e 38c⟩+≡

```
   else if (circe1_params%roots .eq. 500d0) then
      e = GEV500
   else if ((circe1_params%roots .ge. 480d0) .and. (circe1_params%roots .le. 520d0))
      write (msgbuf, 2001) circe1_params%roots, 500d0
      call circem ('MESSAGE', msgbuf)
      e = GEV500
   else if (circe1_params%roots .eq. 800d0) then
      e = GEV800
   else if ((circe1_params%roots .ge. 750d0) .and. (circe1_params%roots .le. 850d0))
      write (msgbuf, 2001) circe1_params%roots, 800d0
      call circem ('MESSAGE', msgbuf)
      e = GEV800
   else if (circe1_params%roots .eq. 1000d0) then
      e = TEV1
   else if ((circe1_params%roots .ge. 900d0) .and. (circe1_params%roots .le. 1100d0))
      write (msgbuf, 2001) circe1_params%roots, 1000d0
      call circem ('MESSAGE', msgbuf)
      e = TEV1
   else if (circe1_params%roots .eq. 1600d0) then
```

```
          e = TEV16
       else if ((circe1_params%roots .ge. 1500d0) .and. (circe1_params%roots .le. 1700d0)
          write (msgbuf, 2001) circe1_params%roots, 1600d0
          call circem ('MESSAGE', msgbuf)
          e = TEV16
```
Uses circem 86e.

39a ⟨*Map* roots *to* e 38c⟩+≡
```
       else
          call circem ('ERROR', &
              'only ROOTS = 350, 500, 800, 1000 and 1600GeV available')
          call circem ('MESSAGE', 'falling back to 500GeV')
          e = GEV500
       endif
```
Uses circem 86e.

39b ⟨*Update version 1 derived parameters in circe1 parameters* 37d⟩+≡
```
       if (xa1lum(e,circe1_params%acc,r) .lt. 0d0) then
          write (msgbuf, 2002) circe1_params%roots, accnam(circe1_params%acc), r
          call circem ('ERROR', msgbuf)
          call circem ('MESSAGE', 'falling back to 500GeV')
          e = GEV500
       end if
  ⟨Log energy mapping 39d⟩
```
Uses circem 86e.

39c ⟨formats *for* circes 38d⟩+≡
```
  2002 format ('energy ', F6.1, ' not available for ', A6,' in revison ', I2)
```

39d ⟨*Log energy mapping* 39d⟩≡
```
       if (circe1_params%chat .ge. 2) then
          if (e .ge. GEV090) then
             write (msgbuf, 2003) circe1_params%roots, e
             call circem ('MESSAGE', msgbuf)
          else if (elo .ge. GEV090 .and. ehi .ge. GEV090) then
             write (msgbuf, 2013) circe1_params%roots, elo, ehi
             call circem ('MESSAGE', msgbuf)
          end if
       endif
```
Uses circem 86e.

39e ⟨formats *for* circes 38d⟩+≡
```
  2003 format ('mapping energy ', F6.1, ' to energy index ', I2)
  2013 format ('mapping energy ', F6.1, ' to energy indices ', I2, ' and ', I2)
```

The energies 250 GeV, 1.2 TeV and 1.5 TeV were entered late into the game by the SLAC people. And, of course, 200 GeV and 230 GeV only appeared even much later

39f ⟨*Local variables for* circes 33b⟩+≡
```
      integer, parameter :: EINVAL = -2
      integer, parameter :: GEV090 = -1
      integer, parameter :: GEV170 = 0
      integer, parameter :: GEV350 = 1
```

```
integer, parameter :: GEV500 = 2
integer, parameter :: GEV800 = 3
integer, parameter :: TEV1   = 4
integer, parameter :: TEV16  = 5
integer, parameter :: GEV250 = 6
integer, parameter :: TEV12  = 7
integer, parameter :: TEV15  = 8
integer, parameter :: GEV200 = 9
integer, parameter :: GEV230 = 10
integer, parameter :: A1NEGY = 5
integer, parameter :: A1NREV = 5
integer :: i
```

40a ⟨*8-byte aligned part of circe1 parameters* 32d⟩+≡

```
real(kind=double) :: lumi
real(kind=double) :: a1(0:7)
```

40b ⟨*Update version 1 derived parameters in circe1 parameters* 37d⟩+≡

```
  circe1_params%lumi = xa1lum (e,circe1_params%acc,r)
  do i = 0, 7
     circe1_params%a1(i) = xa1(i,e,circe1_params%acc,r)
  end do
```

40c ⟨*Local variables for* circes 33b⟩+≡

```
real(kind=double), dimension(A1NEGY,NACC,0:A1NREV), save :: xa1lum = 0
real(kind=double), dimension(0:7,A1NEGY,NACC,0:A1NREV), save :: xa1 = 0
```

Uses NACC 13b.

Revision 1. The mother of all revisions.

40d ⟨*Initializations for* circes 35b⟩+≡

```
xa1lum(GEV500,SBAND,1) = 5.212299E+01
xa1(0:7,GEV500,SBAND,1) = (/ &
   .39192E+00, .66026E+00, .11828E+02,-.62543E+00, &
   .52292E+00,-.69245E+00, .14983E+02, .65421E+00 /)
xa1lum(GEV500,TESLA,1) = 6.066178E+01
xa1(0:7,GEV500,TESLA,1) = (/ &
   .30196E+00, .12249E+01, .21423E+02,-.57848E+00, &
   .68766E+00,-.69788E+00, .23121E+02, .78399E+00 /)
xa1lum(GEV500,XBAND,1) = 5.884699E+01
xa1(0:7,GEV500,XBAND,1) = (/ &
   .48594E+00, .52435E+00, .83585E+01,-.61347E+00, &
   .30703E+00,-.68804E+00, .84109E+01, .44312E+00 /)
```

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

40e ⟨*Initializations for* circes 35b⟩+≡

```
xa1lum(TEV1,SBAND,1) = 1.534650E+02
xa1(0:7,TEV1,SBAND,1) = (/ &
    .24399E+00, .87464E+00, .66751E+01,-.56808E+00, &
    .59295E+00,-.68921E+00, .94232E+01, .83351E+00 /)
xa1lum(TEV1,TESLA,1) = 1.253381E+03
xa1(0:7,TEV1,TESLA,1) = (/ &
    .39843E+00, .70097E+00, .11602E+02,-.61061E+00, &
```

```
        .40737E+00,-.69319E+00, .14800E+02, .51382E+00 /)
    xa1lum(TEV1,XBAND,1) = 1.901783E+02
    xa1(0:7,TEV1,XBAND,1) = (/ &
        .32211E+00,   .61798E+00,   .28298E+01,  -.54644E+00, &
        .45674E+00,  -.67301E+00,   .41703E+01,   .74536E+00 /)
```

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

Unavailable

41a ⟨*Initializations for* circes 35b⟩+≡

```
    xa1lum(GEV350,1:NACC,1) = NACC * (-1d0)
    xa1lum(GEV800,1:NACC,1) = NACC * (-1d0)
```

Uses NACC 13b.

Unavailable as well

41b ⟨*Initializations for* circes 35b⟩+≡

```
    xa1lum(GEV500,SBNDEE:NACC,1) = 4 * (-1d0)
    xa1lum(TEV1,SBNDEE:NACC,1) = 4 * (-1d0)
```

Uses NACC 13b and SBNDEE 13a.

No 1.6TeV parameters in this revision

41c ⟨*Initializations for* circes 35b⟩+≡

```
    xa1lum(TEV16,1:NACC,1) = 7 * (-1d0)
```

Uses NACC 13b.

41d ⟨*Public subroutines* 31a⟩+≡

```
  public :: circel
```

Uses circel 41e.

41e ⟨*Module subroutines* 31b⟩+≡

```
  subroutine circel (l)
    real(kind=double), intent(out) :: l
    l = circe1_params%lumi
  end subroutine circel
```

Defines:
circel, used in chunks 41e, 12a, and 41d.

41f ⟨*Public subroutines* 31a⟩+≡

```
  public :: circee
```

Uses circee 41g.

41g ⟨*Module subroutines* 31b⟩+≡

```
  function circee (x1, x2)
    real(kind=double) :: x1, x2
    real(kind=double) :: circee
    real(kind=double) :: d1, d2
```

⟨*Initialization check* 32g⟩

```
    circee = -1.0
    if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
```

⟨*Calculate version 1 of the* e^+e^- *distribution* 42a⟩

⟨else *handle invalid versions* 37b⟩

```
  end function circee
```

Defines:
circee, used in chunks 14–16, 31b, 41f, and 42a.
Uses d1 16a and d2 16c.

The first version of the parametrization is factorized

$$D^{\alpha 1\rho}_{p_1 p_2}(x_1, x_2, s) = d^{\alpha 1\rho}_{p_1}(x_1) d^{\alpha 1\rho}_{p_2}(x_2) \tag{15}$$

where the distributions are

$$d^{\alpha 1\rho}_{e^\pm}(x) = a^{\alpha\rho}_0 \delta(1-x) + a^{\alpha\rho}_1 x^{a^{\alpha\rho}_2}(1-x)^{a^{\alpha\rho}_3} \tag{16}$$

$$d_\gamma(x) = a^{\alpha\rho}_4 x^{a^{\alpha\rho}_5}(1-x)^{a^{\alpha\rho}_6} \tag{17}$$

42a ⟨*Calculate version 1 of the* e^+e^- *distribution* 42a⟩≡

```
        if (x1 .eq. 1d0) then
           d1 = circe1_params%a1(0)
        elseif (x1 .lt. 1d0 .and. x1 .gt. 0d0) then
           d1 = circe1_params%a1(1) * x1**circe1_params%a1(2) * (1d0 - x1)**circe1_params%
        elseif (x1 .eq. -1d0) then
           d1 = 1d0 - circe1_params%a1(0)
        else
           d1 = 0d0
        endif
        if (x2 .eq. 1d0) then
           d2 = circe1_params%a1(0)
        elseif (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
           d2 = circe1_params%a1(1) * x2**circe1_params%a1(2) * (1d0 - x2)**circe1_params%
        elseif (x2 .eq. -1d0) then
           d2 = 1d0 - circe1_params%a1(0)
        else
           d2 = 0d0
        endif
        circee = d1 * d2
```

Uses circee 41g, d1 16a, and d2 16c.

42b ⟨*Public subroutines* 31a⟩+≡

```
    public :: circeg
```

Uses circeg 42c.

42c ⟨*Module subroutines* 31b⟩+≡

```
    function circeg (x1, x2)
      real(kind=double) :: x1, x2
      real(kind=double) :: circeg
      real(kind=double) :: d1, d2
  ⟨Initialization check 32g⟩
      circeg = -1.0
      if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
  ⟨Calculate version 1 of the e±γ distribution 43a⟩
  ⟨else handle invalid versions 37b⟩
    end function circeg
```

Defines:
circeg, used in chunks 14, 31b, 42b, and 43a.
Uses d1 16a and d2 16c.

43a ⟨*Calculate version 1 of the $e^{\pm}\gamma$ distribution* 43a⟩≡

```
      if (x1 .eq. 1d0) then
         d1 = circe1_params%a1(0)
      else if (x1 .lt. 1d0 .and. x1 .gt. 0d0) then
         d1 = circe1_params%a1(1) * x1**circe1_params%a1(2) * (1d0 - x1)**circe1_params%a1
      else if (x1 .eq. -1d0) then
         d1 = 1d0 - circe1_params%a1(0)
      else
         d1 = 0d0
      end if
      if (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
         d2 = circe1_params%a1(4) * x2**circe1_params%a1(5) * (1d0 - x2)**circe1_params%a1
      else if (x2 .eq. -1d0) then
         d2 = circe1_params%a1(7)
      else
         d2 = 0d0
      end if
      circeg = d1 * d2
```

Uses `circeg` 42c, `d1` 16a, and `d2` 16c.

43b ⟨*Public subroutines* 31a⟩+≡

```
    public :: circgg
```

Uses `circgg` 43c.

43c ⟨*Module subroutines* 31b⟩+≡

```
    function circgg (x1, x2)
      real(kind=double) :: x1, x2
      real(kind=double) :: circgg
      real(kind=double) :: d1, d2
  ⟨Initialization check 32g⟩
      circgg = -1.0
      if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
  ⟨Calculate version 1 of the γγ distribution 43d⟩
  ⟨else handle invalid versions 37b⟩
    end function circgg
```

Defines:
`circgg`, used in chunks 14, 31b, 43, and 81a.
Uses `d1` 16a and `d2` 16c.

43d ⟨*Calculate version 1 of the $\gamma\gamma$ distribution* 43d⟩≡

```
        if (x1 .lt. 1d0 .and. x1 .gt. 0d0) then
           d1 = circe1_params%a1(4) * x1**circe1_params%a1(5) * (1d0 - x1)**circe1_params%
        elseif (x1 .eq. -1d0) then
           d1 = circe1_params%a1(7)
        else
           d1 = 0d0
        endif
        if (x2 .lt. 1d0 .and. x2 .gt. 0d0) then
           d2 = circe1_params%a1(4) * x2**circe1_params%a1(5) * (1d0 - x2)**circe1_params%
        elseif (x2 .eq. -1d0) then
           d2 = circe1_params%a1(7)
```

```
        else
           d2 = 0d0
        endif
        circgg = d1 * d2
```

Uses circgg 43c, d1 16a, and d2 16c.

Revision 2. New Tesla parameters, including 350 GeV and 800 GeV.

44a ⟨*Initializations for* circes 35b⟩+≡

```
      xa1lum(GEV500,SBAND,2) = .31057E+02
      xa1(0:7,GEV500,SBAND,2) = (/ &
           .38504E+00, .79723E+00, .14191E+02,-.60456E+00, &
           .53411E+00,-.68873E+00, .15105E+02, .65151E+00 /)
      xa1lum(TEV1,SBAND,2) = .24297E+03
      xa1(0:7,TEV1,SBAND,2) = (/ &
           .24374E+00, .89466E+00, .70242E+01,-.56754E+00, &
           .60910E+00,-.68682E+00, .96083E+01, .83985E+00 /)
      xa1lum(GEV350,TESLA,2) = .73369E+02
      xa1(0:7,GEV350,TESLA,2) = (/ &
           .36083E+00, .12819E+01, .37880E+02,-.59492E+00, &
           .69109E+00,-.69379E+00, .40061E+02, .65036E+00 /)
      xa1lum(GEV500,TESLA,2) = .10493E+03
      xa1(0:7,GEV500,TESLA,2) = (/ &
           .29569E+00, .11854E+01, .21282E+02,-.58553E+00, &
           .71341E+00,-.69279E+00, .24061E+02, .77709E+00 /)
      xa1lum(GEV800,TESLA,2) = .28010E+03
      xa1(0:7,GEV800,TESLA,2) = (/ &
           .22745E+00, .11265E+01, .10483E+02,-.55711E+00, &
           .69579E+00,-.69068E+00, .13093E+02, .89605E+00 /)
      xa1lum(TEV1,TESLA,2) = .10992E+03
      xa1(0:7,TEV1,TESLA,2) = (/ &
           .40969E+00, .66105E+00, .11972E+02,-.62041E+00, &
           .40463E+00,-.69354E+00, .14669E+02, .51281E+00 /)
      xa1lum(GEV500,XBAND,2) = .35689E+02
      xa1(0:7,GEV500,XBAND,2) = (/ &
           .48960E+00, .46815E+00, .75249E+01,-.62769E+00, &
           .30341E+00,-.68754E+00, .85545E+01, .43453E+00 /)
      xa1lum(TEV1,XBAND,2) = .11724E+03
      xa1(0:7,TEV1,XBAND,2) =  (/ &
           .31939E+00, .62415E+00, .30763E+01,-.55314E+00, &
           .45634E+00,-.67089E+00, .41529E+01, .73807E+00 /)
```

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

Unavailable

44b ⟨*Initializations for* circes 35b⟩+≡

```
      xa1lum(GEV350,SBAND,2) = -1d0
      xa1lum(GEV350,XBAND,2) = -1d0
      xa1lum(GEV800,SBAND,2) = -1d0
      xa1lum(GEV800,XBAND,2) = -1d0
```

Uses SBAND 13a and XBAND 13a.

Unavailable as well

44c ⟨*Initializations for* `circes` 35b⟩+≡

```
xa1lum(GEV350,SBNDEE:NACC,2) = 4 * (-1d0)
xa1lum(GEV500,SBNDEE:NACC,2) = 4 * (-1d0)
xa1lum(GEV800,SBNDEE:NACC,2) = 4 * (-1d0)
xa1lum(TEV1,SBNDEE:NACC,2) = 4 * (-1d0)
```

Uses `NACC` 13b and `SBNDEE` 13a.

No 1.6TeV parameters in this revision

45a ⟨*Initializations for* `circes` 35b⟩+≡

```
xa1lum(TEV16,1:NACC,2) = 7 * (-1d0)
```

Uses `NACC` 13b.

Revision 3. Features:

- improved error estimates.
- cleaner fitting procedure, including delta function pieces.

45b ⟨*Initializations for* `circes` 35b⟩+≡

```
xa1lum(GEV500,SBAND,3) = .31469E+02
xa1(0:7,GEV500,SBAND,3) = (/ &
    .38299E+00, .72035E+00, .12618E+02,-.61611E+00, &
    .51971E+00,-.68960E+00, .15066E+02, .63784E+00 /)
xa1lum(TEV1,SBAND,3) = .24566E+03
xa1(0:7,TEV1,SBAND,3) = (/ &
    .24013E+00, .95763E+00, .69085E+01,-.55151E+00, &
    .59497E+00,-.68622E+00, .94494E+01, .82158E+00 /)
xa1lum(GEV350,TESLA,3) = .74700E+02
xa1(0:7,GEV350,TESLA,3) = (/ &
    .34689E+00, .12484E+01, .33720E+02,-.59523E+00, &
    .66266E+00,-.69524E+00, .38488E+02, .63775E+00 /)
xa1lum(GEV500,TESLA,3) = .10608E+03
xa1(0:7,GEV500,TESLA,3) = (/ &
    .28282E+00, .11700E+01, .19258E+02,-.58390E+00, &
    .68777E+00,-.69402E+00, .23638E+02, .75929E+00 /)
xa1lum(GEV800,TESLA,3) = .28911E+03
xa1(0:7,GEV800,TESLA,3) = (/ &
    .21018E+00, .12039E+01, .96763E+01,-.54024E+00, &
    .67220E+00,-.69083E+00, .12733E+02, .87355E+00 /)
xa1lum(TEV1,TESLA,3) = .10936E+03
xa1(0:7,TEV1,TESLA,3) = (/ &
    .41040E+00, .68099E+00, .11610E+02,-.61237E+00, &
    .40155E+00,-.69073E+00, .14698E+02, .49989E+00 /)
xa1lum(GEV500,XBAND,3) = .36145E+02
xa1(0:7,GEV500,XBAND,3) = (/ &
    .51285E+00, .45812E+00, .75135E+01,-.62247E+00, &
    .30444E+00,-.68530E+00, .85519E+01, .43062E+00 /)
xa1lum(TEV1,XBAND,3) = .11799E+03
xa1(0:7,TEV1,XBAND,3) = (/ &
    .31241E+00, .61241E+00, .29938E+01,-.55848E+00, &
    .44801E+00,-.67116E+00, .41119E+01, .72753E+00 /)
```

Uses `SBAND` 13a, `TESLA` 13a, and `XBAND` 13a.

Still unavailable

46a ⟨*Initializations for* `circes` 35b⟩+≡
```
xa1lum(GEV350,SBAND,3) = -1d0
xa1lum(GEV350,XBAND,3) = -1d0
xa1lum(GEV800,SBAND,3) = -1d0
xa1lum(GEV800,XBAND,3) = -1d0
```
Uses `SBAND` 13a and `XBAND` 13a.

Unavailable as well

46b ⟨*Initializations for* `circes` 35b⟩+≡
```
xa1lum(GEV350,SBNDEE:NACC,3) = 4 * (-1d0)
xa1lum(GEV500,SBNDEE:NACC,3) = 4 * (-1d0)
xa1lum(GEV800,SBNDEE:NACC,3) = 4 * (-1d0)
xa1lum(TEV1,SBNDEE:NACC,3) = 4 * (-1d0)
```
Uses `NACC` 13b and `SBNDEE` 13a.

No 1.6TeV parameters in this revision

46c ⟨*Initializations for* `circes` 35b⟩+≡
```
xa1lum(TEV16,1:NACC,3) = 7 * (-1d0)
```
Uses `NACC` 13b.

Revision 4. Features:

- a bug in `Guinea-Pig`'s synchrotron radiation spectrum has been fixed.

46d ⟨*Initializations for* `circes` 35b⟩+≡
```
xa1lum(GEV500,SBAND,4) = .31528E+02
xa1(0:7,GEV500,SBAND,4) = (/ &
    .38169E+00, .73949E+00, .12543E+02,-.61112E+00, &
    .51256E+00,-.69009E+00, .14892E+02, .63314E+00 /)
xa1lum(TEV1,SBAND,4) = .24613E+03
xa1(0:7,TEV1,SBAND,4) = (/ &
    .24256E+00, .94117E+00, .66775E+01,-.55160E+00, &
    .57484E+00,-.68891E+00, .92271E+01, .81162E+00 /)
xa1lum(GEV350,TESLA,4) = .74549E+02
xa1(0:7,GEV350,TESLA,4) = (/ &
    .34120E+00, .12230E+01, .32932E+02,-.59850E+00, &
    .65947E+00,-.69574E+00, .38116E+02, .63879E+00 /)
xa1lum(GEV500,TESLA,4) = .10668E+03
xa1(0:7,GEV500,TESLA,4) = (/ &
    .28082E+00, .11074E+01, .18399E+02,-.59118E+00, &
    .68880E+00,-.69375E+00, .23463E+02, .76073E+00 /)
xa1lum(GEV800,TESLA,4) = .29006E+03
xa1(0:7,GEV800,TESLA,4) = (/ &
    .21272E+00, .11443E+01, .92564E+01,-.54657E+00, &
    .66799E+00,-.69137E+00, .12498E+02, .87571E+00 /)
xa1lum(TEV1,TESLA,4) = .11009E+03
xa1(0:7,TEV1,TESLA,4) = (/ &
    .41058E+00, .64745E+00, .11271E+02,-.61996E+00, &
    .39801E+00,-.69150E+00, .14560E+02, .49924E+00 /)
xa1lum(GEV500,XBAND,4) = .36179E+02
xa1(0:7,GEV500,XBAND,4) = (/ &
```

```
        .51155E+00, .43313E+00, .70446E+01,-.63003E+00, &
        .29449E+00,-.68747E+00, .83489E+01, .42458E+00 /)
    xa1lum(TEV1,XBAND,4) = .11748E+03
    xa1(0:7,TEV1,XBAND,4) = (/ &
        .32917E+00, .54322E+00, .28493E+01,-.57959E+00, &
        .39266E+00,-.68217E+00, .38475E+01, .68478E+00 /)
```
Uses SBAND 13a, TESLA 13a, and XBAND 13a.

Still unavailable

47a ⟨*Initializations for* circes 35b⟩+≡
```
    xa1lum(GEV350,SBAND,4) = -1d0
    xa1lum(GEV350,XBAND,4) = -1d0
    xa1lum(GEV800,SBAND,4) = -1d0
    xa1lum(GEV800,XBAND,4) = -1d0
```
Uses SBAND 13a and XBAND 13a.

Unavailable as well

47b ⟨*Initializations for* circes 35b⟩+≡
```
    xa1lum(GEV350,SBNDEE:NACC,4) = 4 * (-1d0)
    xa1lum(GEV500,SBNDEE:NACC,4) = 4 * (-1d0)
    xa1lum(GEV800,SBNDEE:NACC,4) = 4 * (-1d0)
    xa1lum(TEV1,SBNDEE:NACC,4) = 4 * (-1d0)
```
Uses NACC 13b and SBNDEE 13a.

No 1.6TeV parameters in this revision

47c ⟨*Initializations for* circes 35b⟩+≡
```
  xa1lum(TEV16,1:NACC,4) = 7 * (-1d0)
```
Uses NACC 13b.

Revision 5. Features:

- a bug in Guinea-Pig has been fixed.
- updated parameter sets

47d ⟨*Initializations for* circes 35b⟩+≡
```
    xa1lum(GEV350,SBAND,5) = 0.21897E+02
    xa1(0:7,GEV350,SBAND,5) = (/ &
        0.57183E+00, 0.53877E+00, 0.19422E+02,-0.63064E+00, &
        0.49112E+00,-0.69109E+00, 0.24331E+02, 0.52718E+00 /)
    xa1lum(GEV500,SBAND,5) = 0.31383E+02
    xa1(0:7,GEV500,SBAND,5) = (/ &
        0.51882E+00, 0.49915E+00, 0.11153E+02,-0.63017E+00, &
        0.50217E+00,-0.69113E+00, 0.14935E+02, 0.62373E+00 /)
    xa1lum(GEV800,SBAND,5) = 0.95091E+02
    xa1(0:7,GEV800,SBAND,5) = (/ &
        0.47137E+00, 0.46150E+00, 0.56562E+01,-0.61758E+00, &
        0.46863E+00,-0.68897E+00, 0.85876E+01, 0.67577E+00 /)
    xa1lum(TEV1,SBAND,5) = 0.11900E+03
    xa1(0:7,TEV1,SBAND,5) = (/ &
        0.43956E+00, 0.45471E+00, 0.42170E+01,-0.61180E+00, &
        0.48711E+00,-0.68696E+00, 0.67145E+01, 0.74551E+00 /)
    xa1lum(TEV16,SBAND,5) = 0.11900E+03
```

```
    xa1(0:7,TEV16,SBAND,5) = (/ &
        0.43956E+00, 0.45471E+00, 0.42170E+01,-0.61180E+00, &
        0.48711E+00,-0.68696E+00, 0.67145E+01, 0.74551E+00 /)
    xa1lum(GEV350,TESLA,5) = 0.97452E+02
    xa1(0:7,GEV350,TESLA,5) = (/ &
        0.39071E+00, 0.84996E+00, 0.17614E+02,-0.60609E+00, &
        0.73920E+00,-0.69490E+00, 0.28940E+02, 0.77286E+00 /)
    xa1lum(GEV500,TESLA,5) = 0.10625E+03
    xa1(0:7,GEV500,TESLA,5) = (/ &
        0.42770E+00, 0.71457E+00, 0.15284E+02,-0.61664E+00, &
        0.68166E+00,-0.69208E+00, 0.24165E+02, 0.73806E+00 /)
    xa1lum(GEV800,TESLA,5) = 0.17086E+03
    xa1(0:7,GEV800,TESLA,5) = (/ &
        0.36025E+00, 0.69118E+00, 0.76221E+01,-0.59440E+00, &
        0.71269E+00,-0.69077E+00, 0.13117E+02, 0.91780E+00 /)
    xa1lum(TEV1,TESLA,5) = 0.21433E+03
    xa1(0:7,TEV1,TESLA,5) = (/ &
        0.33145E+00, 0.67075E+00, 0.55438E+01,-0.58468E+00, &
        0.72503E+00,-0.69084E+00, 0.99992E+01, 0.10112E+01 /)
    xa1lum(TEV16,TESLA,5) = 0.34086E+03
    xa1(0:7,TEV16,TESLA,5) = (/ &
        0.49058E+00, 0.42609E+00, 0.50550E+01,-0.61867E+00, &
        0.39225E+00,-0.68916E+00, 0.75514E+01, 0.58754E+00 /)
    xa1lum(GEV350,XBAND,5) = 0.31901E+02
    xa1(0:7,GEV350,XBAND,5) = (/ &
        0.65349E+00, 0.31752E+00, 0.94342E+01,-0.64291E+00, &
        0.30364E+00,-0.68989E+00, 0.11446E+02, 0.40486E+00 /)
    xa1lum(GEV500,XBAND,5) = 0.36386E+02
    xa1(0:7,GEV500,XBAND,5) = (/ &
        0.65132E+00, 0.28728E+00, 0.69853E+01,-0.64440E+00, &
        0.28736E+00,-0.68758E+00, 0.83227E+01, 0.41492E+00 /)
    xa1lum(GEV800,XBAND,5) = 0.10854E+03
    xa1(0:7,GEV800,XBAND,5) = (/ &
        0.49478E+00, 0.36221E+00, 0.30116E+01,-0.61548E+00, &
        0.39890E+00,-0.68418E+00, 0.45183E+01, 0.67243E+00 /)
    xa1lum(TEV1,XBAND,5) = 0.11899E+03
    xa1(0:7,TEV1,XBAND,5) = (/ &
        0.49992E+00, 0.34299E+00, 0.26184E+01,-0.61584E+00, &
        0.38450E+00,-0.68342E+00, 0.38589E+01, 0.67408E+00 /)
    xa1lum(TEV16,XBAND,5) = 0.13675E+03
    xa1(0:7,TEV16,XBAND,5) = (/ &
        0.50580E+00, 0.30760E+00, 0.18339E+01,-0.61421E+00, &
        0.35233E+00,-0.68315E+00, 0.26708E+01, 0.67918E+00 /)
```

Uses `SBAND` 13a, `TESLA` 13a, and `XBAND` 13a.

Revision 0. Features:

- e^-e^- mode

48 ⟨*Initializations for* `circes` 35b⟩+≡

```
    xa1lum(GEV500,SBNDEE,0) = .92914E+01
```

```
    xa1(0:7,GEV500,SBNDEE,0) = (/ &
        .34866E+00, .78710E+00, .10304E+02,-.59464E+00, &
        .40234E+00,-.69741E+00, .20645E+02, .47274E+00 /)
    xa1lum(TEV1,SBNDEE,0) = .45586E+02
    xa1(0:7,TEV1,SBNDEE,0) = (/ &
        .21084E+00, .99168E+00, .54407E+01,-.52851E+00, &
        .47493E+00,-.69595E+00, .12480E+02, .64027E+00 /)
    xa1lum(GEV350,TESLEE,0) = .15175E+02
    xa1(0:7,GEV350,TESLEE,0) = (/ &
        .33093E+00, .11137E+01, .25275E+02,-.59942E+00, &
        .49623E+00,-.70403E+00, .60188E+02, .44637E+00 /)
    xa1lum(GEV500,TESLEE,0) = .21622E+02
    xa1(0:7,GEV500,TESLEE,0) = (/ &
        .27175E+00, .10697E+01, .14858E+02,-.58418E+00, &
        .50824E+00,-.70387E+00, .36129E+02, .53002E+00 /)
    xa1lum(GEV800,TESLEE,0) = .43979E+02
    xa1(0:7,GEV800,TESLEE,0) = (/ &
        .22994E+00, .10129E+01, .81905E+01,-.55751E+00, &
        .46551E+00,-.70461E+00, .19394E+02, .58387E+00 /)
    xa1lum(TEV1,TESLEE,0) = .25465E+02
    xa1(0:7,TEV1,TESLEE,0) = (/ &
        .37294E+00, .67522E+00, .87504E+01,-.60576E+00, &
        .35095E+00,-.69821E+00, .18526E+02, .42784E+00 /)
    xa1lum(GEV500,XBNDEE,0) = .13970E+02
    xa1(0:7,GEV500,XBNDEE,0) = (/ &
        .47296E+00, .46800E+00, .58897E+01,-.61689E+00, &
        .27181E+00,-.68923E+00, .10087E+02, .37462E+00 /)
    xa1lum(TEV1,XBNDEE,0) = .41056E+02
    xa1(0:7,TEV1,XBNDEE,0) = (/ &
        .27965E+00, .74816E+00, .27415E+01,-.50491E+00, &
        .38320E+00,-.67945E+00, .47506E+01, .62218E+00 /)
```

Uses SBNDEE 13a, TESLEE 13a, and XBNDEE 13a.

Still unavailable

49a ⟨*Initializations for* circes 35b⟩+≡

```
    xa1lum(GEV350,SBNDEE,0) = -1d0
    xa1lum(GEV350,XBNDEE,0) = -1d0
    xa1lum(GEV800,SBNDEE,0) = -1d0
    xa1lum(GEV800,XBNDEE,0) = -1d0
```

Uses SBNDEE 13a and XBNDEE 13a.

49b ⟨*Initializations for* circes 35b⟩+≡

```
    xa1lum(GEV500,SBAND,0) = .31528E+02
    xa1(0:7,GEV500,SBAND,0) = (/ &
        .38169E+00, .73949E+00, .12543E+02,-.61112E+00, &
        .51256E+00,-.69009E+00, .14892E+02, .63314E+00 /)
    xa1lum(TEV1,SBAND,0) = .24613E+03
    xa1(0:7,TEV1,SBAND,0) = (/ &
        .24256E+00, .94117E+00, .66775E+01,-.55160E+00, &
        .57484E+00,-.68891E+00, .92271E+01, .81162E+00 /)
    xa1lum(GEV350,TESLA,0) = .74549E+02
```

```
xa1(0:7,GEV350,TESLA,0) = (/ &
    .34120E+00, .12230E+01, .32932E+02,-.59850E+00, &
    .65947E+00,-.69574E+00, .38116E+02, .63879E+00 /)
xa1lum(GEV500,TESLA,0) = .10668E+03
xa1(0:7,GEV500,TESLA,0) = (/ &
    .28082E+00, .11074E+01, .18399E+02,-.59118E+00, &
    .68880E+00,-.69375E+00, .23463E+02, .76073E+00 /)
xa1lum(GEV800,TESLA,0) = .29006E+03
xa1(0:7,GEV800,TESLA,0) = (/ &
    .21272E+00, .11443E+01, .92564E+01,-.54657E+00, &
    .66799E+00,-.69137E+00, .12498E+02, .87571E+00 /)
xa1lum(TEV1,TESLA,0) = .11009E+03
xa1(0:7,TEV1,TESLA,0) = (/ &
    .41058E+00, .64745E+00, .11271E+02,-.61996E+00, &
    .39801E+00,-.69150E+00, .14560E+02, .49924E+00 /)
xa1lum(GEV500,XBAND,0) = .36179E+02
xa1(0:7,GEV500,XBAND,0) = (/ &
    .51155E+00, .43313E+00, .70446E+01,-.63003E+00, &
    .29449E+00,-.68747E+00, .83489E+01, .42458E+00 /)
xa1lum(TEV1,XBAND,0) = .11748E+03
xa1(0:7,TEV1,XBAND,0) = (/ &
    .32917E+00, .54322E+00, .28493E+01,-.57959E+00, &
    .39266E+00,-.68217E+00, .38475E+01, .68478E+00 /)
```

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

Still unavailable

50a ⟨*Initializations for* circes 35b⟩+≡

```
xa1lum(GEV350,SBAND,0) = -1d0
xa1lum(GEV350,XBAND,0) = -1d0
xa1lum(GEV800,SBAND,0) = -1d0
xa1lum(GEV800,XBAND,0) = -1d0
```

Uses SBAND 13a and XBAND 13a.

6.2.2 Version 2

50b ⟨*Version 2 has been retired* 50b⟩≡

```
call circem ('PANIC', '**********************************')
call circem ('PANIC', '* version 2 has been retired,    *')
call circem ('PANIC', '* please use version 1 instead! *')
call circem ('PANIC', '**********************************')
return
```

Uses circem 86e.

6.2.3 Versions 3 and 4

50c ⟨*Update version 3 and 4 derived parameters in circe1 parameters* 50c⟩≡

```
if (circe1_params%rev .eq. 0) then
   r = 0
elseif (circe1_params%rev .ge. 19970417) then
   r = 5
```

```
          if (ver34 .eq. 3) then
             call circem ('WARNING', 'version 3 retired after 97/04/17')
             call circem ('MESSAGE', 'falling back to version 4')
          end if
       else if (circe1_params%rev .ge. 19961022) then
          r = ver34
          if ((circe1_params%roots .ne. 800d0) .or. (circe1_params%acc .ne. TESLA)) then
             call circem ('ERROR', 'versions 3 and 4 before 97/04/17')
             call circem ('ERROR', 'apply to TESLA at 800 GeV only')
             call circem ('MESSAGE', 'falling back to TESLA at 800GeV')
             circe1_params%acc = TESLA
             e = GEV800
          end if
       else if (circe1_params%rev .lt. 19961022) then
          call circem ('ERROR', &
            'no revision of versions 3 and 4 available before 96/10/22')
          call circem ('MESSAGE', 'falling back to default')
          r = 5
       end if
  ⟨Log revision mapping 38a⟩
```
Uses TESLA 13a and circem 86e.

51a ⟨*Update version 3 and 4 derived parameters in circe1 parameters* 50c⟩+≡
```
  ⟨Map roots to e 38c⟩
       if (xa3lum(e,circe1_params%acc,r) .lt. 0d0) then
          write (msgbuf, 2002) circe1_params%roots, accnam(circe1_params%acc), r
          call circem ('ERROR', msgbuf)
          call circem ('MESSAGE', 'falling back to 500GeV')
          e = GEV500
       endif
  ⟨Log energy mapping 39d⟩
```
Uses circem 86e.

51b ⟨*Local variables for* circes 33b⟩+≡
```
      integer, parameter :: A3NEGY = 5, A3NREV = 5
```

51c ⟨*Update version 3 and 4 derived parameters in circe1 parameters* 50c⟩+≡
```
       circe1_params%lumi = xa3lum (e,circe1_params%acc,r)
       do i = 0, 7
          circe1_params%a1(i) = xa3(i,e,circe1_params%acc,r)
       end do
```

51d ⟨*Local variables for* circes 33b⟩+≡
```
      real, dimension(A3NEGY,NACC,0:A3NREV), save :: xa3lum = -1
      real, dimension(0:7,A3NEGY,NACC,0:A3NREV), save :: xa3 = 0
```
Uses NACC 13b.

Revisions 3 & 4. The mother of all revisions.

51e ⟨*Initializations for* circes 35b⟩+≡
```
      xa3lum(GEV800,TESLA,3) = .17196E+03
      xa3(0:7,GEV800,TESLA,3) = (/ &
          .21633E+00, .11333E+01, .95928E+01,-.55095E+00, &
```

```
    .73044E+00,-.69101E+00, .12868E+02, .94737E+00 /)
xa3lum(GEV800,TESLA, 4) = .16408E+03
xa3(0:7,GEV800,TESLA, 4) = (/ &
    .41828E+00, .72418E+00, .14137E+02,-.61189E+00, &
    .36697E+00,-.69205E+00, .17713E+02, .43583E+00 /)
```

Uses TESLA 13a.

Revision 5.

52a ⟨*Initializations for* circes 35b⟩+≡

```
xa3lum(GEV350,TESLA,5) = 0.66447E+02
xa3(0:7,GEV350,TESLA,5) = (/ &
    0.69418E+00, 0.50553E+00, 0.48430E+02,-0.63911E+00, &
    0.34074E+00,-0.69533E+00, 0.55502E+02, 0.29397E+00 /)
xa3lum(GEV500,TESLA,5) = 0.95241E+02
xa3(0:7,GEV500,TESLA,5) = (/ &
    0.64882E+00, 0.45462E+00, 0.27103E+02,-0.64535E+00, &
    0.35101E+00,-0.69467E+00, 0.33658E+02, 0.35024E+00 /)
xa3lum(GEV800,TESLA,5) = 0.16974E+03
xa3(0:7,GEV800,TESLA,5) = (/ &
    0.58706E+00, 0.43771E+00, 0.13422E+02,-0.63804E+00, &
    0.35541E+00,-0.69467E+00, 0.17528E+02, 0.43051E+00 /)
xa3lum(TEV1,TESLA,5) = 0.21222E+03
xa3(0:7,TEV1,TESLA,5) = (/ &
    0.55525E+00, 0.42577E+00, 0.96341E+01,-0.63587E+00, &
    0.36448E+00,-0.69365E+00, 0.13161E+02, 0.47715E+00 /)
xa3lum(TEV16,TESLA,5) = 0.34086E+03
xa3(0:7,TEV16,TESLA,5) = (/ &
    0.49058E+00, 0.42609E+00, 0.50550E+01,-0.61867E+00, &
    0.39225E+00,-0.68916E+00, 0.75514E+01, 0.58754E+00 /)
```

Uses TESLA 13a.

Revision 0. Currently identical to revision 5.

52b ⟨*Initializations for* circes 35b⟩+≡

```
xa3lum(GEV350,TESLA,0) = 0.66447E+02
xa3(0:7,GEV350,TESLA,0) = (/ &
    0.69418E+00, 0.50553E+00, 0.48430E+02,-0.63911E+00, &
    0.34074E+00,-0.69533E+00, 0.55502E+02, 0.29397E+00 /)
xa3lum(GEV500,TESLA,0) = 0.95241E+02
xa3(0:7,GEV500,TESLA,0) = (/ &
    0.64882E+00, 0.45462E+00, 0.27103E+02,-0.64535E+00, &
    0.35101E+00,-0.69467E+00, 0.33658E+02, 0.35024E+00 /)
xa3lum(GEV800,TESLA,0) = 0.16974E+03
xa3(0:7,GEV800,TESLA,0) = (/ &
    0.58706E+00, 0.43771E+00, 0.13422E+02,-0.63804E+00, &
    0.35541E+00,-0.69467E+00, 0.17528E+02, 0.43051E+00 /)
xa3lum(TEV1,TESLA,0) = 0.21222E+03
xa3(0:7,TEV1,TESLA,0) = (/ &
    0.55525E+00, 0.42577E+00, 0.96341E+01,-0.63587E+00, &
    0.36448E+00,-0.69365E+00, 0.13161E+02, 0.47715E+00 /)
xa3lum(TEV16,TESLA,0) = 0.34086E+03
xa3(0:7,TEV16,TESLA,0) = (/ &
```

```
          0.49058E+00, 0.42609E+00, 0.50550E+01,-0.61867E+00, &
          0.39225E+00,-0.68916E+00, 0.75514E+01, 0.58754E+00 /)
```

Uses TESLA 13a.

6.2.4 Version 5

53a ⟨*Update version 5 derived parameters in circe1 parameters* 53a⟩≡

```
        if (circe1_params%rev .eq. 0) then
           r = 0
        elseif (circe1_params%rev .ge. 19980505) then
           r = 1
        elseif (circe1_params%rev .lt. 19980505) then
           call circem ('ERROR', &
            'no revision of version 5 available before 98/05/05')
           call circem ('MESSAGE', 'falling back to default')
           r = 1
        endif
  ⟨Log revision mapping 38a⟩
```

Uses circem 86e.

53b ⟨*Update version 5 derived parameters in circe1 parameters* 53a⟩+≡

```
        if (circe1_params%acc .ne. TESLA) then
           call circem ('ERROR', 'versions 5 applies to TESLA only')
           circe1_params%acc = TESLA
        end if
  ⟨Map roots to e 38c⟩
        if (xa5lum(e,circe1_params%acc,r) .lt. 0d0) then
           write (msgbuf, 2002) circe1_params%roots, accnam(circe1_params%acc), r
           call circem ('ERROR', msgbuf)
           call circem ('MESSAGE', 'falling back to 500GeV')
           e = GEV500
        endif
  ⟨Log energy mapping 39d⟩
```

Uses TESLA 13a and circem 86e.

53c ⟨*Local variables for* circes 33b⟩+≡

```
      integer, parameter :: A5NEGY = 5, A5NREV = 1
```

53d ⟨*Update version 5 derived parameters in circe1 parameters* 53a⟩+≡

```
        circe1_params%lumi = xa5lum (e,circe1_params%acc,r)
        do i = 0, 7
           circe1_params%a1(i) = xa5(i,e,circe1_params%acc,r)
        end do
```

53e ⟨*Local variables for* circes 33b⟩+≡

```
      real, dimension(A5NEGY,NACC,0:A5NREV), save :: xa5lum
      real, dimension(0:7,A5NEGY,NACC,0:A5NREV), save :: xa5
```

Uses NACC 13b.

Revision 1. The mother of all revisions. Note that $3.3980 \cdot 10^{34}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1} = 2.4099 \cdot 10^{34}\,\mathrm{m}^{-2} \cdot 2820 \cdot 5\,\mathrm{s}^{-1}$ and $3.5936 \cdot 10^{34}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1} = 2.6619 \cdot 10^{34}\,\mathrm{m}^{-2} \cdot 4500 \cdot 3\,\mathrm{s}^{-1}$. This unit conversion is missing in *all* earlier versions, unfortunately.

54a ⟨*Initializations for* `circes` 35b⟩+≡

```
      xa5lum(GEV350,TESLA,1) = -1.0
      xa5lum(GEV500,TESLA,1) = 0.33980E+03
      xa5(0:7,GEV500,TESLA,1) = (/ &
         0.49808E+00, 0.54613E+00, 0.12287E+02,-0.62756E+00, &
         0.42817E+00,-0.69120E+00, 0.17067E+02, 0.51143E+00 /)
      xa5lum(GEV800,TESLA,1) = 0.35936E+03
      xa5(0:7,GEV800,TESLA,1) = (/ &
         0.58751E+00, 0.43128E+00, 0.13324E+02,-0.64006E+00, &
         0.30682E+00,-0.69235E+00, 0.16815E+02, 0.37078E+00 /)
      xa5lum(TEV1, TESLA,1) = -1.0
      xa5lum(TEV16,TESLA,1) = -1.0
```

Uses `TESLA` 13a.

Revision 0. Currently identical to revision 1.

54b ⟨*Initializations for* `circes` 35b⟩+≡

```
      xa5lum(GEV350,TESLA,0) = -1.0
      xa5lum(GEV500,TESLA,0) = 0.33980E+03
      xa5(0:7,GEV500,TESLA,0) = (/ &
         0.49808E+00, 0.54613E+00, 0.12287E+02,-0.62756E+00, &
         0.42817E+00,-0.69120E+00, 0.17067E+02, 0.51143E+00 /)
      xa5lum(GEV800,TESLA,0) = 0.35936E+03
      xa5(0:7,GEV800,TESLA,0) = (/ &
         0.58751E+00, 0.43128E+00, 0.13324E+02,-0.64006E+00, &
         0.30682E+00,-0.69235E+00, 0.16815E+02, 0.37078E+00 /)
      xa5lum(TEV1, TESLA,0) = -1.0
      xa5lum(TEV16,TESLA,0) = -1.0
```

Uses `TESLA` 13a.

6.2.5 Version 6

54c ⟨*Update version 6 derived parameters in circe1 parameters* 54c⟩≡

```
        if (circe1_params%rev .eq. 0) then
           r = 0
        else if (circe1_params%rev .ge. 19990415) then
           r = 1
        else if (circe1_params%rev .lt. 19990415) then
           call circem ('ERROR', &
             'no revision of version 6 available before 1999/04/15')
           call circem ('MESSAGE', 'falling back to default')
           r = 1
        end if
  ⟨Log revision mapping 38a⟩
```

Uses `circem` 86e.

54d ⟨*Update version 6 derived parameters in circe1 parameters* 54c⟩+≡

```
        if (circe1_params%acc .ne. TESLA) then
```

```
          call circem ('ERROR', 'versions 6 applies to TESLA only')
          circe1_params%acc = TESLA
       end if
    ⟨Map roots to e at low energies 55a⟩
       if (xa6lum(e,circe1_params%acc,r) .lt. 0d0) then
          write (msgbuf, 2002) circe1_params%roots, accnam(circe1_params%acc), r
          call circem ('ERROR', msgbuf)
          call circem ('MESSAGE', 'falling back to 500GeV')
          e = GEV500
       endif
    ⟨Log energy mapping 39d⟩
```
Uses TESLA 13a and circem 86e.

55a ⟨*Map* roots *to* e *at low energies* 55a⟩≡
```
       if (circe1_params%roots .eq.  90d0) then
          e = GEV090
       elseif ((circe1_params%roots .ge. 85d0) .and. (circe1_params%roots .le. 95d0)) the
          write (msgbuf, 2001) circe1_params%roots, 90d0
          call circem ('MESSAGE', msgbuf)
          e = GEV090
       elseif (circe1_params%roots .eq. 170d0) then
          e = GEV170
       elseif ((circe1_params%roots .ge. 160d0) .and. (circe1_params%roots .le. 180d0)) t
          write (msgbuf, 2001) circe1_params%roots, 170d0
          call circem ('MESSAGE', msgbuf)
          e = GEV170
       elseif (circe1_params%roots .eq. 350d0) then
          e = GEV350
       elseif ((circe1_params%roots .ge. 340d0) .and. (circe1_params%roots .le. 370d0)) t
          write (msgbuf, 2001) circe1_params%roots, 350d0
          call circem ('MESSAGE', msgbuf)
          e = GEV350
       elseif (circe1_params%roots .eq. 500d0) then
          e = GEV500
       elseif ((circe1_params%roots .ge. 480d0) .and. (circe1_params%roots .le. 520d0)) t
          write (msgbuf, 2001) circe1_params%roots, 500d0
          call circem ('MESSAGE', msgbuf)
          e = GEV500
       else
          call circem ('ERROR', &
              'only ROOTS = 90, 170, 350, and 500GeV available')
          call circem ('MESSAGE', 'falling back to 500GeV')
          e = GEV500
       endif
```
Uses circem 86e.

55b ⟨*Local variables for* circes 33b⟩+≡
```
     integer, parameter :: A6NEGY = 2, A6NREV = 1
```

55c ⟨*Update version 6 derived parameters in circe1 parameters* 54c⟩+≡
```
       circe1_params%lumi = xa6lum (e,circe1_params%acc,r)
```

```
      do i = 0, 7
         circe1_params%a1(i) = xa6(i,e,circe1_params%acc,r)
      end do
```

56a ⟨*Local variables for* `circes` 33b⟩+≡

```
  real, dimension(GEV090:A6NEGY,NACC,0:A6NREV), save :: xa6lum
  real, dimension(0:7,GEV090:A6NEGY,NACC,0:A6NREV), save :: xa6
```

Uses `NACC` 13b.

Revision 1. The mother of all revisions.

56b ⟨*Initializations for* `circes` 35b⟩+≡

```
  xa6lum(GEV090,TESLA,1) = 0.62408E+02
  xa6(0:7,GEV090,TESLA,1) = (/ &
     0.72637E+00, 0.75534E+00, 0.18180E+03,-0.63426E+00, &
     0.36829E+00,-0.69653E+00, 0.18908E+03, 0.22157E+00 /)
  xa6lum(GEV170,TESLA,1) = 0.11532E+02
  xa6(0:7,GEV170,TESLA,1) = (/ &
     0.65232E+00, 0.67249E+00, 0.66862E+02,-0.63315E+00, &
     0.38470E+00,-0.69477E+00, 0.75120E+02, 0.30162E+00 /)
  xa6lum(GEV350,TESLA,1) = 0.24641E+03
  xa6(0:7,GEV350,TESLA,1) = (/ &
     0.54610E+00, 0.59105E+00, 0.20297E+02,-0.62747E+00, &
     0.41588E+00,-0.69188E+00, 0.26345E+02, 0.43818E+00 /)
  xa6lum(GEV500,TESLA,1) = 0.30340E+03
  xa6(0:7,GEV500,TESLA,1) = (/ &
     0.52744E+00, 0.52573E+00, 0.13895E+02,-0.63145E+00, &
     0.40824E+00,-0.69150E+00, 0.18645E+02, 0.47585E+00 /)
```

Uses `TESLA` 13a.

Revision 0. Currently identical to revision 1.

56c ⟨*Initializations for* `circes` 35b⟩+≡

```
  xa6lum(GEV090,TESLA,0) = 0.62408E+02
  xa6(0:7,GEV090,TESLA,0) = (/ &
     0.72637E+00, 0.75534E+00, 0.18180E+03,-0.63426E+00, &
     0.36829E+00,-0.69653E+00, 0.18908E+03, 0.22157E+00 /)
  xa6lum(GEV170,TESLA,0) = 0.11532E+02
  xa6(0:7,GEV170,TESLA,0) = (/ &
     0.65232E+00, 0.67249E+00, 0.66862E+02,-0.63315E+00, &
     0.38470E+00,-0.69477E+00, 0.75120E+02, 0.30162E+00 /)
  xa6lum(GEV350,TESLA,0) = 0.24641E+03
  xa6(0:7,GEV350,TESLA,0) = (/ &
     0.54610E+00, 0.59105E+00, 0.20297E+02,-0.62747E+00, &
     0.41588E+00,-0.69188E+00, 0.26345E+02, 0.43818E+00 /)
  xa6lum(GEV500,TESLA,0) = 0.30340E+03
  xa6(0:7,GEV500,TESLA,0) = (/ &
     0.52744E+00, 0.52573E+00, 0.13895E+02,-0.63145E+00, &
     0.40824E+00,-0.69150E+00, 0.18645E+02, 0.47585E+00 /)
```

Uses `TESLA` 13a.

6.2.6 Version 7

56d ⟨*Update version 7 derived parameters in circe1 parameters* 56d⟩≡

```
        if (circe1_params%rev .eq. 0) then
           r = 0
        elseif (circe1_params%rev .ge. 20000426) then
           r = 1
        elseif (circe1_params%rev .lt. 20000426) then
           call circem ('ERROR', &
            'no revision of version 7 available before 2000/04/26')
           call circem ('MESSAGE', 'falling back to default')
           r = 1
        endif
  ⟨Log revision mapping 38a⟩
```
Uses circem 86e.

57a ⟨*Update version 7 derived parameters in circe1 parameters* 56d⟩+≡
```
        if (circe1_params%acc .ne. TESLA .and. circe1_params%acc .ne. JLCNLC) then
           call circem ('ERROR', &
                        'version 7 applies to TESLA and JLCNLC only')
           call circem ('ERROR', 'falling back to TESLA')
           circe1_params%acc = TESLA
        end if
  ⟨Linearly interpolate energies 57d⟩
  ⟨Log energy mapping 39d⟩
```
Uses JLCNLC 13a, TESLA 13a, and circem 86e.

57b ⟨formats *for* circes 38d⟩+≡
```
  2004 format ('energy ', F6.1, 'GeV too low, using spectrum for ', F6.1, 'GeV')
  2005 format ('energy ', F6.1, 'GeV too high, using spectrum for ', F6.1, 'GeV')
  2006 format ('energy ', F6.1, 'GeV interpolated between ', F6.1, ' and ', F6.1, 'GeV')
```

57c ⟨*Local variables for* circes 33b⟩+≡
```
      real(kind=double) :: eloval, ehival
      real(kind=double), parameter :: DELTAE = 0.5d0
```

The rules are as follows: XBAND has 500 GeV and 1 TeV, TESLA has 500 GeV and 800 TeV. Low energy TESLA will be added.

57d ⟨*Linearly interpolate energies* 57d⟩≡
```
        e = GEV090 - 1
        elo = e
        ehi = e
        if (circe1_params%acc .eq. TESLA) then
           if (circe1_params%roots .lt.  90d0 - DELTAE) then
              write (msgbuf, 2004) circe1_params%roots, 90d0
              call circem ('MESSAGE', msgbuf)
              e = GEV090
           elseif (abs (circe1_params%roots-090d0) .le. DELTAE) then
              e = GEV090
           elseif (circe1_params%roots .lt. 170d0 - DELTAE) then
              write (msgbuf, 2005) circe1_params%roots, 170d0
              call circem ('MESSAGE', msgbuf)
              e = GEV170
           elseif (abs (circe1_params%roots-170d0) .le. DELTAE) then
              e = GEV170
```

```
   elseif (circe1_params%roots .lt. 350d0-DELTAE) then
      write (msgbuf, 2006) circe1_params%roots, 170d0, 350d0
      call circem ('MESSAGE', msgbuf)
      elo = GEV170
      ehi = GEV350
      eloval = 170d0
      ehival = 350d0
   elseif (abs (circe1_params%roots-350d0) .le. DELTAE) then
      e = GEV350
   elseif (circe1_params%roots .lt. 500d0 - DELTAE) then
      write (msgbuf, 2006) circe1_params%roots, 350d0, 500d0
      call circem ('MESSAGE', msgbuf)
      elo = GEV350
      ehi = GEV500
      eloval = 350d0
      ehival = 500d0
   elseif (abs (circe1_params%roots-500d0) .le. DELTAE) then
      e = GEV500
   elseif (circe1_params%roots .lt. 800d0 - DELTAE) then
      write (msgbuf, 2006) circe1_params%roots, 500d0, 800d0
      call circem ('MESSAGE', msgbuf)
      elo = GEV500
      ehi = GEV800
      eloval = 500d0
      ehival = 800d0
   elseif (abs (circe1_params%roots-800d0) .le. DELTAE) then
      e = GEV800
   else
      write (msgbuf, 2005) circe1_params%roots, 800d0
      call circem ('MESSAGE', msgbuf)
      e = GEV800
   endif
elseif (circe1_params%acc .eq. XBAND) then
   if (circe1_params%roots .lt.  500d0 - DELTAE) then
      write (msgbuf, 2004) circe1_params%roots, 500d0
      call circem ('MESSAGE', msgbuf)
      e = GEV500
   elseif (abs (circe1_params%roots-500d0) .le. DELTAE) then
      e = GEV500
   elseif (circe1_params%roots .lt. 1000d0 - DELTAE) then
      write (msgbuf, 2006) circe1_params%roots, 500d0, 1000d0
      call circem ('MESSAGE', msgbuf)
      elo = GEV500
      ehi = TEV1
      eloval =  500d0
      ehival = 1000d0
   elseif (abs (circe1_params%roots-1000d0) .le. DELTAE) then
      e = TEV1
   else
      write (msgbuf, 2005) circe1_params%roots, 1000d0
```

```
            call circem ('MESSAGE', msgbuf)
            e = TEV1
         endif
      endif
```
Uses TESLA 13a, XBAND 13a, and circem 86e.

59a ⟨*Local variables for* circes 33b⟩+≡
```
      integer, parameter :: A7NEGY = TEV1, A7NREV = 1
```
Note that ew *must not* interpolate a1(0) and a1(7) because they depend non-linearly on the other parameters!

59b ⟨*Update version 7 derived parameters in circe1 parameters* 56d⟩+≡
```
        if (e .ge. GEV090) then
           circe1_params%lumi = xa7lum(e,circe1_params%acc,r)
           do i = 0, 7
              circe1_params%a1(i) = xa7(i,e,circe1_params%acc,r)
           end do
        else if (elo .ge. GEV090 .and. ehi .ge. GEV090) then
           circe1_params%lumi = ((circe1_params%roots-eloval)*xa7lum(ehi,circe1_params%acc
               + (ehival-circe1_params%roots)*xa7lum(elo,circe1_params%acc,r)) / (ehival -
           do i = 1, 6
              circe1_params%a1(i) = ((circe1_params%roots-eloval)*xa7(i,ehi,circe1_params%
                  + (ehival-circe1_params%roots)*xa7(i,elo,circe1_params%acc,r)) / (ehival
           end do
           circe1_params%a1(0) = 1d0 - circe1_params%a1(1) * beta(circe1_params%a1(2)+1d0,
           circe1_params%a1(7) = circe1_params%a1(4) * beta(circe1_params%a1(5)+1d0,circe1
        endif
```
Uses beta 105.

59c ⟨*Local variables for* circes 33b⟩+≡
```
      real, dimension(GEV090:A7NEGY,NACC,0:A7NREV), save :: xa7lum
      real, dimension(0:7,GEV090:A7NEGY,NACC,0:A7NREV), save :: xa7
```
Uses NACC 13b.

Revision 1. The mother of all revisions.

59d ⟨*Initializations for* circes 35b⟩+≡
```
      xa7lum(GEV090,TESLA,1) = 0.62408E+02
      xa7(0:7,GEV090,TESLA,1) = (/ &
         0.72637E+00, 0.75534E+00, 0.18180E+03,-0.63426E+00, &
         0.36829E+00,-0.69653E+00, 0.18908E+03, 0.22157E+00 /)
      xa7lum(GEV170,TESLA,1) = 0.11532E+02
      xa7(0:7,GEV170,TESLA,1) = (/ &
         0.65232E+00, 0.67249E+00, 0.66862E+02,-0.63315E+00, &
         0.38470E+00,-0.69477E+00, 0.75120E+02, 0.30162E+00 /)
      xa7lum(GEV350,TESLA,1) = 0.24641E+03
      xa7(0:7,GEV350,TESLA,1) = (/ &
         0.54610E+00, 0.59105E+00, 0.20297E+02,-0.62747E+00, &
         0.41588E+00,-0.69188E+00, 0.26345E+02, 0.43818E+00 /)
      xa7lum(GEV500,TESLA,1) = 0.34704E+03
      xa7(0:7,GEV500,TESLA,1) = (/ &
         0.51288E+00, 0.49025E+00, 0.99716E+01,-0.62850E+00, &
         0.41048E+00,-0.69065E+00, 0.13922E+02, 0.51902E+00 /)
```

```
xa7lum(GEV800,TESLA,1) = 0.57719E+03
xa7(0:7,GEV800,TESLA,1) = (/ &
   0.52490E+00, 0.42573E+00, 0.69069E+01,-0.62649E+00, &
   0.32380E+00,-0.68958E+00, 0.93819E+01, 0.45671E+00 /)
xa7lum(TEV1,TESLA,1) = -1.0
```

Uses TESLA 13a.

60a ⟨*Initializations for* circes 35b⟩+≡

```
xa7lum(GEV090,JLCNLC,1) = -1.0
xa7lum(GEV170,JLCNLC,1) = -1.0
xa7lum(GEV350,JLCNLC,1) = -1.0
xa7lum(GEV500,JLCNLC,1) = 0.63039E+02
xa7(0:7,GEV500,JLCNLC,1) = (/ &
    0.58967E+00, 0.34035E+00, 0.63631E+01,-0.63683E+00, &
    0.33383E+00,-0.68803E+00, 0.81005E+01, 0.48702E+00 /)
xa7lum(TEV1,JLCNLC,1) = 0.12812E+03
xa7(0:7,TEV1,JLCNLC,1) = (/ &
    0.50222E+00, 0.33773E+00, 0.25681E+01,-0.61711E+00, &
    0.36826E+00,-0.68335E+00, 0.36746E+01, 0.65393E+00 /)
```

Uses JLCNLC 13a.

Revision 0.

60b ⟨*Initializations for* circes 35b⟩+≡

```
xa7lum(GEV090,TESLA,0) = 0.62408E+02
xa7(0:7,GEV090,TESLA,0) = (/ &
    0.72637E+00, 0.75534E+00, 0.18180E+03,-0.63426E+00, &
    0.36829E+00,-0.69653E+00, 0.18908E+03, 0.22157E+00 /)
xa7lum(GEV170,TESLA,0) = 0.11532E+02
xa7(0:7,GEV170,TESLA,0) = (/ &
    0.65232E+00, 0.67249E+00, 0.66862E+02,-0.63315E+00, &
    0.38470E+00,-0.69477E+00, 0.75120E+02, 0.30162E+00 /)
xa7lum(GEV350,TESLA,0) = 0.24641E+03
xa7(0:7,GEV350,TESLA,0) = (/ &
    0.54610E+00, 0.59105E+00, 0.20297E+02,-0.62747E+00, &
    0.41588E+00,-0.69188E+00, 0.26345E+02, 0.43818E+00 /)
xa7lum(GEV500,TESLA,0) = 0.34704E+03
xa7(0:7,GEV500,TESLA,0) = (/ &
    0.51288E+00, 0.49025E+00, 0.99716E+01,-0.62850E+00, &
    0.41048E+00,-0.69065E+00, 0.13922E+02, 0.51902E+00 /)
xa7lum(GEV800,TESLA,0) = 0.57719E+03
xa7(0:7,GEV800,TESLA,0) = (/ &
    0.52490E+00, 0.42573E+00, 0.69069E+01,-0.62649E+00, &
    0.32380E+00,-0.68958E+00, 0.93819E+01, 0.45671E+00 /)
xa7lum(TEV1,TESLA,0) = -1.0
```

Uses TESLA 13a.

60c ⟨*Initializations for* circes 35b⟩+≡

```
xa7lum(GEV090,JLCNLC,0) = -1.0
xa7lum(GEV170,JLCNLC,0) = -1.0
xa7lum(GEV350,JLCNLC,0) = -1.0
xa7lum(GEV500,JLCNLC,0) = 0.63039E+02
```

```
      xa7(0:7,GEV500,JLCNLC,0) = (/ &
          0.58967E+00, 0.34035E+00, 0.63631E+01,-0.63683E+00, &
          0.33383E+00,-0.68803E+00, 0.81005E+01, 0.48702E+00 /)
      xa7lum(TEV1,JLCNLC,0) = 0.12812E+03
      xa7(0:7,TEV1,JLCNLC,0) = (/ &
          0.50222E+00, 0.33773E+00, 0.25681E+01,-0.61711E+00, &
          0.36826E+00,-0.68335E+00, 0.36746E+01, 0.65393E+00 /)
```

Uses JLCNLC 13a.

6.2.7 Version 8

61a ⟨*Update version 8 derived parameters in circe1 parameters* 61a⟩≡

```
        if (circe1_params%rev .eq. 0) then
           r = 0
        elseif (circe1_params%rev .ge. 20010617) then
           r = 1
        elseif (circe1_params%rev .lt. 20010617) then
           call circem ('ERROR', &
            'no revision of version 8 available before 2001/06/17')
           call circem ('MESSAGE', 'falling back to default')
           r = 1
        endif
  ⟨Log revision mapping 38a⟩
```

Uses circem 86e.

61b ⟨*Update version 8 derived parameters in circe1 parameters* 61a⟩+≡

```
        if (circe1_params%acc .eq. NLCH) then
           circe1_params%acc = JLCNLC
        end if
        if (circe1_params%acc .ne. JLCNLC) then
           call circem ('ERROR', &
                        'version 8 applies to JLCNLC (NLC H) only')
           call circem ('ERROR', 'falling back to JLCNLC')
           circe1_params%acc = JLCNLC
        end if
  ⟨Linearly interpolate energies 57d⟩
  ⟨Log energy mapping 39d⟩
```

Uses JLCNLC 13a and circem 86e.

61c ⟨*Local variables for* circes 33b⟩+≡

```
      integer, parameter :: A8NEGY = TEV1, A8NREV = 1
```

Note that ew *must not* interpolate a1(0) and a1(7) because they depend non-linearly on the other parameters!

61d ⟨*Update version 8 derived parameters in circe1 parameters* 61a⟩+≡

```
        if (e .ge. GEV090) then
           circe1_params%lumi = xa8lum(e,circe1_params%acc,r)
           do i = 0, 7
              circe1_params%a1(i) = xa8(i,e,circe1_params%acc,r)
           end do
        elseif (elo .ge. GEV090 .and. ehi .ge. GEV090) then
```

```
          circe1_params%lumi = ((circe1_params%roots-eloval)*xa8lum(ehi,circe1_params%acc
              + (ehival-circe1_params%roots)*xa8lum(elo,circe1_params%acc,r)) / (ehival -
          do i = 1, 6
             circe1_params%a1(i) = ((circe1_params%roots-eloval)*xa8(i,ehi,circe1_params%
                 + (ehival-circe1_params%roots)*xa8(i,elo,circe1_params%acc,r)) / (ehival
          end do
          circe1_params%a1(0) = 1d0 - circe1_params%a1(1) * beta(circe1_params%a1(2)+1d0,
          circe1_params%a1(7) = circe1_params%a1(4) * beta(circe1_params%a1(5)+1d0,circe1
       endif
```

Uses beta 105.

62a ⟨*Local variables for* circes 33b⟩+≡

```
      real, dimension(GEV090:A8NEGY,NACC,0:A8NREV), save :: xa8lum
      real, dimension(0:7,GEV090:A8NEGY,NACC,0:A8NREV), save :: xa8
```

Uses NACC 13b.

Revision 1. The mother of all revisions.

62b ⟨*Initializations for* circes 35b⟩+≡

```
      xa8lum(GEV090,TESLA,1) = -1.0
      xa8lum(GEV170,TESLA,1) = -1.0
      xa8lum(GEV350,TESLA,1) = -1.0
      xa8lum(GEV500,TESLA,1) = -1.0
      xa8lum(GEV800,TESLA,1) = -1.0
      xa8lum(TEV1,  TESLA,1) = -1.0
```

Uses TESLA 13a.

62c ⟨*Initializations for* circes 35b⟩+≡

```
      xa8lum(GEV090,JLCNLC,1) = -1.0
      xa8lum(GEV170,JLCNLC,1) = -1.0
      xa8lum(GEV350,JLCNLC,1) = -1.0
      xa8lum(GEV500,JLCNLC,1) = 0.239924E+03
      xa8(0:7,GEV500,JLCNLC,1) = (/ &
         0.57025E+00, 0.34004E+00, 0.52864E+01,-0.63405E+00, &
         0.31627E+00,-0.68722E+00, 0.69629E+01, 0.47973E+00 /)
      xa8lum(TEV1,JLCNLC,1) = 0.40858E+03
      xa8(0:7,TEV1,JLCNLC,1) = (/ &
         0.52344E+00, 0.31536E+00, 0.25244E+01,-0.62215E+00, &
         0.31935E+00,-0.68424E+00, 0.35877E+01, 0.57315E+00 /)
```

Uses JLCNLC 13a.

Revision 0.

62d ⟨*Initializations for* circes 35b⟩+≡

```
      xa8lum(GEV090,TESLA,0) = -1.0
      xa8lum(GEV170,TESLA,0) = -1.0
      xa8lum(GEV350,TESLA,0) = -1.0
      xa8lum(GEV500,TESLA,0) = -1.0
      xa8lum(GEV800,TESLA,0) = -1.0
      xa8lum(TEV1,  TESLA,0) = -1.0
```

Uses TESLA 13a.

62e ⟨*Initializations for* circes 35b⟩+≡

```
      xa8lum(GEV090,JLCNLC,0) = -1.0
```

```
      xa8lum(GEV170,JLCNLC,0) = -1.0
      xa8lum(GEV350,JLCNLC,0) = -1.0
      xa8lum(GEV500,JLCNLC,0) = 0.239924E+03
      xa8(0:7,GEV500,JLCNLC,0) = (/ &
         0.57025E+00, 0.34004E+00, 0.52864E+01,-0.63405E+00, &
         0.31627E+00,-0.68722E+00, 0.69629E+01, 0.47973E+00 /)
      xa8lum(TEV1,JLCNLC,0) = 0.40858E+03
      xa8(0:7,TEV1,JLCNLC,0) = (/ &
         0.52344E+00, 0.31536E+00, 0.25244E+01,-0.62215E+00, &
         0.31935E+00,-0.68424E+00, 0.35877E+01, 0.57315E+00 /)
```
Uses JLCNLC 13a.

6.2.8 Version 9

63a ⟨*Update version 9 derived parameters in circe1 parameters* 63a⟩≡
```
  if (circe1_params%rev .eq. 0) then
     r = 0
  elseif (circe1_params%rev .ge. 20020328) then
     r = 1
  elseif (circe1_params%rev .lt. 20020328) then
     call circem ('ERROR', &
       'no revision of version 9 available before 2002/03/28')
     call circem ('MESSAGE', 'falling back to default')
     r = 1
  endif
  ⟨Log revision mapping 38a⟩
```
Uses circem 86e.

63b ⟨*Update version 9 derived parameters in circe1 parameters* 63a⟩+≡
```
  if (circe1_params%acc .ne. JLCNLC .and. circe1_params%acc .ne. NLCH) then
     call circem ('ERROR', &
                  'version 9 applies to JLCNLC and NLCH only')
     call circem ('ERROR', 'falling back to JLCNLC')
     circe1_params%acc = JLCNLC
  end if
  if (circe1_params%acc .eq. JLCNLC) then
    ⟨Linearly interpolate energies for JLC/NLC 2002 63c⟩
  else if (circe1_params%acc .eq. NLCH) then
    ⟨Linearly interpolate energies for NLC H 2002 65a⟩
  end if
  ⟨Log energy mapping 39d⟩
```
Uses JLCNLC 13a and circem 86e.

63c ⟨*Linearly interpolate energies for JLC/NLC 2002* 63c⟩≡
```
  e = GEV090 - 1
  elo = e
  ehi = e
  if (circe1_params%roots .lt. 250d0 - DELTAE) then
     write (msgbuf, 2004) circe1_params%roots, 250d0
     call circem ('MESSAGE', msgbuf)
     e = GEV250
```

```
elseif (abs (circe1_params%roots-250d0) .le. DELTAE) then
   e = GEV250
elseif (circe1_params%roots .lt. 500d0 - DELTAE) then
   write (msgbuf, 2006) circe1_params%roots, 250d0, 500d0
   call circem ('MESSAGE', msgbuf)
   elo = GEV250
   ehi = GEV500
   eloval = 250d0
   ehival = 500d0
elseif (abs (circe1_params%roots-500d0) .le. DELTAE) then
   e = GEV500
elseif (circe1_params%roots .lt. 800d0 - DELTAE) then
   write (msgbuf, 2006) circe1_params%roots, 500d0, 800d0
   call circem ('MESSAGE', msgbuf)
   elo = GEV500
   ehi = GEV800
   eloval = 500d0
   ehival = 800d0
elseif (abs (circe1_params%roots-800d0) .le. DELTAE) then
   e = GEV800
elseif (circe1_params%roots .lt. 1000d0 - DELTAE) then
   write (msgbuf, 2006) circe1_params%roots, 800d0, 1000d0
   call circem ('MESSAGE', msgbuf)
   elo = GEV800
   ehi = TEV1
   eloval =  800d0
   ehival = 1000d0
elseif (abs (circe1_params%roots-1000d0) .le. DELTAE) then
   e = TEV1
elseif (circe1_params%roots .lt. 1200d0 - DELTAE) then
   write (msgbuf, 2006) circe1_params%roots, 1000d0, 1200d0
   call circem ('MESSAGE', msgbuf)
   elo = TEV1
   ehi = TEV12
   eloval = 1000d0
   ehival = 1200d0
elseif (abs (circe1_params%roots-1200d0) .le. DELTAE) then
   e = TEV12
elseif (circe1_params%roots .lt. 1500d0 - DELTAE) then
   write (msgbuf, 2006) circe1_params%roots, 1200d0, 1500d0
   call circem ('MESSAGE', msgbuf)
   elo = TEV12
   ehi = TEV15
   eloval = 1200d0
   ehival = 1500d0
elseif (abs (circe1_params%roots-1500d0) .le. DELTAE) then
   e = TEV15
else
   write (msgbuf, 2005) circe1_params%roots, 1500d0
   call circem ('MESSAGE', msgbuf)
```

```
      e = TEV15
   endif
```

Uses circem 86e.

65a ⟨*Linearly interpolate energies for NLC H 2002* 65a⟩≡

```
   e = GEV090 - 1
   elo = e
   ehi = e
   if (circe1_params%roots .lt. 500d0 - DELTAE) then
      write (msgbuf, 2004) circe1_params%roots, 500d0
      call circem ('MESSAGE', msgbuf)
      e = GEV500
   elseif (abs (circe1_params%roots-500d0) .le. DELTAE) then
      e = GEV500
   elseif (circe1_params%roots .lt. 1000d0 - DELTAE) then
      write (msgbuf, 2006) circe1_params%roots, 500d0, 1000d0
      call circem ('MESSAGE', msgbuf)
      elo = GEV500
      ehi = TEV1
      eloval =  500d0
      ehival = 1000d0
   elseif (abs (circe1_params%roots-1000d0) .le. DELTAE) then
      e = TEV1
   elseif (circe1_params%roots .lt. 1500d0 - DELTAE) then
      write (msgbuf, 2006) circe1_params%roots, 1000d0, 1500d0
      call circem ('MESSAGE', msgbuf)
      elo = TEV1
      ehi = TEV15
      eloval = 1000d0
      ehival = 1500d0
   elseif (abs (circe1_params%roots-1500d0) .le. DELTAE) then
      e = TEV15
   else
      write (msgbuf, 2005) circe1_params%roots, 1500d0
      call circem ('MESSAGE', msgbuf)
      e = TEV15
   endif
```

Uses circem 86e.

65b ⟨*Local variables for* circes 33b⟩+≡

```
   integer, parameter ::  A9NEGY = TEV15, A9NREV = 1
```

Note that ew *must not* interpolate a1(0) and a1(7) because they depend non-linearly on the other parameters!

65c ⟨*Update version 9 derived parameters in circe1 parameters* 63a⟩+≡

```
   if (e .ge. GEV090) then
      circe1_params%lumi = xa9lum(e,circe1_params%acc,r)
      do i = 0, 7
         circe1_params%a1(i) = xa9(i,e,circe1_params%acc,r)
      end do
   else if (elo .ge. GEV090 .and. ehi .ge. GEV090) then
```

```
          circe1_params%lumi = ((circe1_params%roots-eloval)*xa9lum(ehi,circe1_params%acc
              + (ehival-circe1_params%roots)*xa9lum(elo,circe1_params%acc,r)) / (ehival -
          do i = 1, 6
             circe1_params%a1(i) = ((circe1_params%roots-eloval)*xa9(i,ehi,circe1_params%
                 + (ehival-circe1_params%roots)*xa9(i,elo,circe1_params%acc,r)) / (ehival
          end do
          circe1_params%a1(0) = 1d0 - circe1_params%a1(1) * beta(circe1_params%a1(2)+1d0,
          circe1_params%a1(7) = circe1_params%a1(4) * beta(circe1_params%a1(5)+1d0,circe1
       end if
```

Uses beta 105.

66a ⟨*Local variables for* circes 33b⟩+≡

```
      real, dimension(GEV090:A9NEGY,NACC,0:A9NREV) :: xa9lum
      real, dimension(0:7,GEV090:A9NEGY,NACC,0:A9NREV) :: xa9
```

Uses NACC 13b.

Revision 1. The mother of all revisions.

66b ⟨*Initializations for* circes 35b⟩+≡

```
      xa9lum(GEV090,TESLA,1) = -1.0
      xa9lum(GEV170,TESLA,1) = -1.0
      xa9lum(GEV350,TESLA,1) = -1.0
      xa9lum(GEV500,TESLA,1) = -1.0
      xa9lum(GEV800,TESLA,1) = -1.0
      xa9lum(TEV1,  TESLA,1) = -1.0
      xa9lum(TEV12, TESLA,1) = -1.0
      xa9lum(TEV15, TESLA,1) = -1.0
      xa9lum(TEV16, TESLA,1) = -1.0
```

Uses TESLA 13a.

66c ⟨*Initializations for* circes 35b⟩+≡

```
      xa9lum(GEV090,JLCNLC,1) = -1.0
      xa9lum(GEV170,JLCNLC,1) = -1.0
      xa9lum(GEV250,JLCNLC,1) = 109.886976
      xa9(0:7,GEV250,JLCNLC,1) = (/ &
          0.65598E+00, 0.34993E+00, 0.13766E+02,-0.64698E+00, &
          0.29984E+00,-0.69053E+00, 0.16444E+02, 0.36060E+00 /)
      xa9lum(GEV350,JLCNLC,1) = -1.0
      xa9lum(GEV500,JLCNLC,1) = 220.806144
      xa9(0:7,GEV500,JLCNLC,1) = (/ &
          0.57022E+00, 0.33782E+00, 0.52811E+01,-0.63540E+00, &
          0.32035E+00,-0.68776E+00, 0.69552E+01, 0.48751E+00 /)
      xa9lum(GEV800,JLCNLC,1) = 304.63488
      xa9(0:7,GEV800,JLCNLC,1) = (/ &
          0.54839E+00, 0.31823E+00, 0.33071E+01,-0.62671E+00, &
          0.31655E+00,-0.68468E+00, 0.45325E+01, 0.53449E+00 /)
      xa9lum(TEV1, JLCNLC,1) = 319.95648
      xa9(0:7,TEV1, JLCNLC,1) = (/ &
          0.56047E+00, 0.29479E+00, 0.28820E+01,-0.62856E+00, &
          0.29827E+00,-0.68423E+00, 0.39138E+01, 0.52297E+00 /)
      xa9lum(TEV12,JLCNLC,1) = 349.90848
      xa9(0:7,TEV12,JLCNLC,1) = (/ &
```

```
    0.56102E+00, 0.28503E+00, 0.24804E+01,-0.62563E+00, &
    0.29002E+00,-0.68376E+00, 0.33854E+01, 0.52736E+00 /)
xa9lum(TEV15,JLCNLC,1) = 363.15648
xa9(0:7,TEV15,JLCNLC,1) = (/ &
    0.57644E+00, 0.26570E+00, 0.22007E+01,-0.62566E+00, &
    0.27102E+00,-0.68283E+00, 0.29719E+01, 0.50764E+00 /)
xa9lum(TEV16,JLCNLC,1) = -1.0
```

Uses JLCNLC 13a.

67a ⟨*Initializations for* circes 35b⟩+≡

```
xa9lum(GEV090,NLCH,1) = -1.0
xa9lum(GEV170,NLCH,1) = -1.0
xa9lum(GEV250,NLCH,1) = -1.0
xa9lum(GEV350,NLCH,1) = -1.0
xa9lum(GEV500,NLCH,1) = 371.4624
xa9(0:7,GEV500,NLCH,1)= (/ &
    0.33933E+00, 0.55165E+00, 0.29138E+01,-0.57341E+00, &
    0.54323E+00,-0.68590E+00, 0.51786E+01, 0.88956E+00 /)
xa9lum(GEV800,NLCH,1) = -1.0
xa9lum(TEV1,NLCH,1) = 516.41856
xa9(0:7,TEV1,NLCH,1)= (/ &
    0.35478E+00, 0.46474E+00, 0.17666E+01,-0.56949E+00, &
    0.49269E+00,-0.68384E+00, 0.31781E+01, 0.91121E+00 /)
xa9lum(TEV12,NLCH,1) = -1.0
xa9lum(TEV15,NLCH,1) = 575.06688
xa9(0:7,TEV15,NLCH,1)= (/ &
    0.38183E+00, 0.40310E+00, 0.13704E+01,-0.57742E+00, &
    0.44548E+00,-0.68341E+00, 0.24956E+01, 0.87448E+00 /)
xa9lum(TEV16,NLCH,  1) = -1.0
```

Revision 0.

67b ⟨*Initializations for* circes 35b⟩+≡

```
xa9lum(GEV090,TESLA,0) = -1.0
xa9lum(GEV170,TESLA,0) = -1.0
xa9lum(GEV350,TESLA,0) = -1.0
xa9lum(GEV500,TESLA,0) = -1.0
xa9lum(GEV800,TESLA,0) = -1.0
xa9lum(TEV1,  TESLA,0) = -1.0
xa9lum(TEV12, TESLA,0) = -1.0
xa9lum(TEV15, TESLA,0) = -1.0
xa9lum(TEV16, TESLA,0) = -1.0
```

Uses TESLA 13a.

67c ⟨*Initializations for* circes 35b⟩+≡

```
xa9lum(GEV090,JLCNLC,0) = -1.0
xa9lum(GEV170,JLCNLC,0) = -1.0
xa9lum(GEV250,JLCNLC,0) = 109.886976
xa9(0:7,GEV250,JLCNLC,0) = (/ &
    0.65598E+00, 0.34993E+00, 0.13766E+02,-0.64698E+00, &
    0.29984E+00,-0.69053E+00, 0.16444E+02, 0.36060E+00 /)
xa9lum(GEV350,JLCNLC,0) = -1.0
```

```
      xa9lum(GEV500,JLCNLC,0) = 220.806144
      xa9(0:7,GEV500,JLCNLC,0) = (/ &
          0.57022E+00, 0.33782E+00, 0.52811E+01,-0.63540E+00, &
          0.32035E+00,-0.68776E+00, 0.69552E+01, 0.48751E+00 /)
      xa9lum(GEV800,JLCNLC,0) = 304.63488
      xa9(0:7,GEV800,JLCNLC,0) = (/ &
          0.54839E+00, 0.31823E+00, 0.33071E+01,-0.62671E+00, &
          0.31655E+00,-0.68468E+00, 0.45325E+01, 0.53449E+00 /)
      xa9lum(TEV1, JLCNLC,0) = 319.95648
      xa9(0:7,TEV1, JLCNLC,0) = (/ &
          0.56047E+00, 0.29479E+00, 0.28820E+01,-0.62856E+00, &
          0.29827E+00,-0.68423E+00, 0.39138E+01, 0.52297E+00 /)
      xa9lum(TEV12,JLCNLC,0) = 349.90848
      xa9(0:7,TEV12,JLCNLC,0) = (/ &
          0.56102E+00, 0.28503E+00, 0.24804E+01,-0.62563E+00, &
          0.29002E+00,-0.68376E+00, 0.33854E+01, 0.52736E+00 /)
      xa9lum(TEV15,JLCNLC,0) = 363.15648
      xa9(0:7,TEV15,JLCNLC,0) = (/ &
          0.57644E+00, 0.26570E+00, 0.22007E+01,-0.62566E+00, &
          0.27102E+00,-0.68283E+00, 0.29719E+01, 0.50764E+00 /)
      xa9lum(TEV16,JLCNLC,0) = -1.0
```

Uses JLCNLC 13a.

68a ⟨*Initializations for* circes 35b⟩+≡

```
      xa9lum(GEV090,NLCH,0) = -1.0
      xa9lum(GEV170,NLCH,0) = -1.0
      xa9lum(GEV250,NLCH,0) = -1.0
      xa9lum(GEV350,NLCH,0) = -1.0
      xa9lum(GEV500,NLCH,0) = 371.4624
      xa9(0:7,GEV500,NLCH,0) = (/ &
          0.33933E+00, 0.55165E+00, 0.29138E+01,-0.57341E+00, &
          0.54323E+00,-0.68590E+00, 0.51786E+01, 0.88956E+00 /)
      xa9lum(GEV800,NLCH,0) = -1.0
      xa9lum(TEV1,NLCH,0)   = 516.41856
      xa9(0:7,TEV1,NLCH,0) = (/ &
          0.35478E+00, 0.46474E+00, 0.17666E+01,-0.56949E+00, &
          0.49269E+00,-0.68384E+00, 0.31781E+01, 0.91121E+00 /)
      xa9lum(TEV12,NLCH,0) = -1.0
      xa9lum(TEV15,NLCH,0) = 575.06688
      xa9(0:7,TEV15,NLCH,0) = (/ &
          0.38183E+00, 0.40310E+00, 0.13704E+01,-0.57742E+00, &
          0.44548E+00,-0.68341E+00, 0.24956E+01, 0.87448E+00 /)
      xa9lum(TEV16,NLCH,0) = -1.0
```

6.2.9 Version 10

68b ⟨*Update version 10 derived parameters in circe1 parameters* 68b⟩≡

```
        if (circe1_params%rev .eq. 0) then
           r = 0
        elseif (circe1_params%rev .ge. 20140305) then
```

```
      r = 1
   elseif (circe1_params%rev .lt. 20140305) then
      call circem ('ERROR', &
       'no revision of version 10 available before 2014/03/05')
      call circem ('MESSAGE', 'falling back to default')
      r = 1
   endif
   ⟨Log revision mapping 38a⟩
```
Uses circem 86e.

69a ⟨*Update version 10 derived parameters in circe1 parameters* 68b⟩+≡
```
   if (circe1_params%acc .ne. ILC) then
      call circem ('ERROR', 'version 10 applies to ILC only')
      call circem ('ERROR', 'falling back to ILC')
      circe1_params%acc = ILC
   end if
   if (circe1_params%acc .eq. ILC) then
     ⟨Linearly interpolate energies for ILC 2013 69b⟩
   end if
   ⟨Log energy mapping 39d⟩
```
Uses ILC 13a and circem 86e.

69b ⟨*Linearly interpolate energies for ILC 2013* 69b⟩≡
```
   e = -EINVAL
   elo = -EINVAL
   ehi = -EINVAL
   if (circe1_params%roots .lt. 200d0 - DELTAE) then
      write (msgbuf, 2004) circe1_params%roots, 200d0
      call circem ('MESSAGE', msgbuf)
      e = GEV200
   elseif (abs (circe1_params%roots-200d0) .le. DELTAE) then
      e = GEV200
   elseif (circe1_params%roots .lt. 230d0 - DELTAE) then
      write (msgbuf, 2006) circe1_params%roots, 200d0, 230d0
      call circem ('MESSAGE', msgbuf)
      elo = GEV200
      ehi = GEV230
      eloval = 200d0
      ehival = 230d0
   elseif (abs (circe1_params%roots-230d0) .le. DELTAE) then
      e = GEV230
   elseif (circe1_params%roots .lt. 250d0 - DELTAE) then
      write (msgbuf, 2006) circe1_params%roots, 230d0, 250d0
      call circem ('MESSAGE', msgbuf)
      elo = GEV230
      ehi = GEV250
      eloval = 230d0
      ehival = 250d0
   elseif (abs (circe1_params%roots-250d0) .le. DELTAE) then
      e = GEV250
   elseif (circe1_params%roots .lt. 350d0 - DELTAE) then
```

```
          write (msgbuf, 2006) circe1_params%roots, 250d0, 350d0
          call circem ('MESSAGE', msgbuf)
          elo = GEV250
          ehi = GEV350
          eloval = 250d0
          ehival = 350d0
       elseif (abs (circe1_params%roots-350d0) .le. DELTAE) then
          e = GEV350
       elseif (circe1_params%roots .lt. 500d0 - DELTAE) then
          write (msgbuf, 2006) circe1_params%roots, 350d0, 500d0
          call circem ('MESSAGE', msgbuf)
          elo = GEV350
          ehi = GEV500
          eloval = 350d0
          ehival = 500d0
       elseif (abs (circe1_params%roots-500d0) .le. DELTAE) then
          e = GEV500
       else
          write (msgbuf, 2005) circe1_params%roots, 500d0
          call circem ('MESSAGE', msgbuf)
          e = GEV500
       endif
```

Uses circem 86e.

70a ⟨*Local variables for* circes 33b⟩+≡

```
    integer, parameter ::  A10NEGY = GEV230, A10NREV = 1
```

Note that ew *must not* interpolate a1(0) and a1(7) because they depend non-linearly on the other parameters!

70b ⟨*Update version 10 derived parameters in circe1 parameters* 68b⟩+≡

```
       if (e .ne. EINVAL) then
          circe1_params%lumi = xa10lum(e,circe1_params%acc,r)
          do i = 0, 7
             circe1_params%a1(i) = xa10(i,e,circe1_params%acc,r)
          end do
       else if (elo .ne. EINVAL .and. ehi .ne. EINVAL) then
          circe1_params%lumi = ((circe1_params%roots-eloval)*xa10lum(ehi,circe1_params%ac
              + (ehival-circe1_params%roots)*xa10lum(elo,circe1_params%acc,r)) / (ehival
          do i = 1, 6
             circe1_params%a1(i) = ((circe1_params%roots-eloval)*xa10(i,ehi,circe1_params
                 + (ehival-circe1_params%roots)*xa10(i,elo,circe1_params%acc,r)) / (ehiva
          end do
          circe1_params%a1(0) = 1d0 - circe1_params%a1(1) * beta(circe1_params%a1(2)+1d0,
          circe1_params%a1(7) = circe1_params%a1(4) * beta(circe1_params%a1(5)+1d0,circe1
       end if
```

Uses beta 105.

70c ⟨*Local variables for* circes 33b⟩+≡

```
    real, dimension(GEV090:A10NEGY,ILC:ILC,0:A10NREV) :: xa10lum
    real, dimension(0:7,GEV090:A10NEGY,ILC:ILC,0:A10NREV) :: xa10
```

Uses ILC 13a.

Revision 1. The mother of all revisions.

71a ⟨*Initializations for* `circes` 35b⟩+≡
```
xa10lum = -1
xa10 = -1
```

71b ⟨*Initializations for* `circes` 35b⟩+≡
```
xa10lum(GEV200,ILC,1) =  56
xa10(:,GEV200,ILC,1) = (/ &
     0.66253E+00,  0.51646E+00,  0.43632E+02, -0.64508E+00, &
     0.35915E+00, -0.69716E+00,  0.51645E+02,  0.32097E+00 /)
xa10lum(GEV230,ILC,1) =  83
xa10(:,GEV230,ILC,1) = (/ &
     0.62360E+00,  0.52780E+00,  0.31915E+02, -0.64171E+00, &
     0.38375E+00, -0.69529E+00,  0.39717E+02,  0.36597E+00 /)
xa10lum(GEV250,ILC,1) =  97
xa10(:,GEV250,ILC,1) = (/ &
     0.59996E+00,  0.52141E+00,  0.26647E+02, -0.64331E+00, &
     0.39186E+00, -0.69687E+00,  0.33764E+02,  0.39669E+00 /)
xa10lum(GEV350,ILC,1) = 100
xa10(:,GEV350,ILC,1) = (/ &
     0.58875E+00,  0.50027E+00,  0.18594E+02, -0.63380E+00, &
     0.38659E+00, -0.69239E+00,  0.23964E+02,  0.42049E+00 /)
xa10lum(GEV500,ILC,1) = 180
xa10(:,GEV500,ILC,1) = (/ &
     0.46755E+00,  0.51768E+00,  0.83463E+01, -0.62311E+00, &
     0.45704E+00, -0.69165E+00,  0.12372E+02,  0.60192E+00 /)
```
Uses `ILC` 13a.

71c ⟨*Initializations for* `circes` 35b⟩+≡

Revision 0 The latest is the default:

71d ⟨*Initializations for* `circes` 35b⟩+≡
```
xa10lum(:,:,0) = xa10lum(:,:,A10NREV)
xa10(:,:,:,0) = xa10(:,:,:,A10NREV)
```

6.3 Special Functions

71e ⟨*Module subroutines* 31b⟩+≡
```
function beta (a, b)
  real(kind=double) :: a, b, beta
  beta = exp (dlogam(a) + dlogam(b) - dlogam(a+b))
end function beta
```
Uses `beta` 105.

71f ⟨*Module subroutines* 31b⟩+≡
```
!!! CERNLIB C304

  function dlogam (x)
    real(kind=double) :: dlogam
    real(kind=double), dimension(7) :: p1, q1, p2, q2, p3, q3
```

```
real(kind=double), dimension(5) :: c, xl
real(kind=double) :: x, y, zero, one, two, half, ap, aq
integer :: i
data ZERO /0.0D0/, ONE /1.0D0/, TWO /2.0D0/, HALF /0.5D0/
data XL /0.0D0,0.5D0,1.5D0,4.0D0,12.0D0/
data p1 /+3.8428736567460D+0, +5.2706893753010D+1, &
         +5.5584045723515D+1, -2.1513513573726D+2, &
         -2.4587261722292D+2, -5.7500893603041D+1, &
         -2.3359098949513D+0/
data q1 /+1.0000000000000D+0, +3.3733047907071D+1, &
         +1.9387784034377D+2, +3.0882954973424D+2, &
         +1.5006839064891D+2, +2.0106851344334D+1, &
         +4.5717420282503D-1/
data p2 /+4.8740201396839D+0, +2.4884525168574D+2, &
         +2.1797366058896D+3, +3.7975124011525D+3, &
         -1.9778070769842D+3, -3.6929834005591D+3, &
         -5.6017773537804D+2/
data q2 /+1.0000000000000D+0, +9.5099917418209D+1, &
         +1.5612045277929D+3, +7.2340087928948D+3, &
         +1.0459576594059D+4, +4.1699415153200D+3, &
         +2.7678583623804D+2/
data p3 /-6.8806240094594D+3, -4.3069969819571D+5, &
         -4.7504594653440D+6, -2.9423445930322D+6, &
         +3.6321804931543D+7, -3.3567782814546D+6, &
         -2.4804369488286D+7/
data q3 /+1.0000000000000D+0, -1.4216829839651D+3, &
         -1.5552890280854D+5, -3.4152517108011D+6, &
         -2.0969623255804D+7, -3.4544175093344D+7, &
         -9.1605582863713D+6/
data c / 1.1224921356561D-1,  7.9591692961204D-2, &
        -1.7087794611020D-3,  9.1893853320467D-1, &
         1.3469905627879D+0/
if (x .le. xl(1)) then
     print *, 'ERROR: DLOGAM non positive argument: ', X
     dlogam = zero
end if
if (x .le. xl(2)) then
  y = x + one
  ap = p1(1)
  aq = q1(1)
  do i = 2, 7
    ap = p1(i) + y * ap
    aq = q1(i) + y * aq
  end do
  y = - log(x) + x * ap / aq
else if (x .le. xl(3)) then
  ap = p1(1)
  aq = q1(1)
  do i = 2, 7
     ap = p1(i) + x * ap
```

```
      aq = q1(i) + x * aq
   end do
   y = (x - one) * ap / aq
 else if (x .le. xl(4)) then
   ap = p2(1)
   aq = q2(1)
   do i = 2, 7
      ap = p2(i) + x * ap
      aq = q2(i) + x * aq
   end do
   y = (x-two) * ap / aq
 else if (x .le. xl(5)) then
   ap = p3(1)
   aq = q3(1)
   do i = 2, 7
      ap = p3(i) + x * ap
      aq = q3(i) + x * aq
   end do
   y = ap / aq
 else
  y = one / x**2
  y = (x-half) * log(x) - x + c(4) + &
      (c(1) + y * (c(2) + y * c(3))) / ((c(5) + y) * x)
 end if
 dlogam = y
end function dlogam
```

6.4 Non-Singular Distributions

73a ⟨*Public subroutines* 31a⟩+≡

```
public :: kirke
```

Uses `kirke` 73b.

73b ⟨*Module subroutines* 31b⟩+≡

```
function kirke (x1, x2, p1, p2)
  real(kind=double) :: x1, x2
  real(kind=double) :: kirke
  integer :: p1, p2
  ⟨Initialization check 32g⟩
  kirke = -1.0
  if (abs(p1) .eq. C1_ELECTRON) then
     if (abs(p2) .eq. C1_ELECTRON) then
        kirke = kirkee (x1, x2)
     else if (p2 .eq. C1_PHOTON) then
        kirke = kirkeg (x1, x2)
     end if
  else if (p1 .eq. C1_PHOTON) then
     if (abs(p2) .eq. C1_ELECTRON) then
        kirke = kirkeg (x2, x1)
```

```
        else if (p2 .eq. C1_PHOTON) then
           kirke = kirkgg (x1, x2)
        end if
     endif
  end function kirke
```

Defines:
`kirke`, used in chunk 73a.
Uses `C1_ELECTRON` 11b, `C1_PHOTON` 11b, `kirkee` 74b, `kirkeg` 76c, and `kirkgg` 77a.

74a ⟨*Public subroutines* 31a⟩+≡

```
  public :: kirkee
```

Uses `kirkee` 74b.

74b ⟨*Module subroutines* 31b⟩+≡

```
  function kirkee (x1, x2)
    real(kind=double) :: x1, x2
    real(kind=double) :: kirkee
    real(kind=double) :: d1, d2
    ⟨Initialization check 32g⟩
    kirkee = -1.0
    if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
       ⟨Calculate version 1 of the non-singular e+e- distribution 75c⟩
    ⟨else handle invalid versions 37b⟩
  end function kirkee
```

Defines:
`kirkee`, used in chunks 17a and 73–75.
Uses `d1` 16a and `d2` 16c.

74c ⟨*8-byte aligned part of circe1 parameters* 32d⟩+≡

```
    real(kind=double) :: elect0, gamma0
```

$$\int_{1-\epsilon}^{1^+} \mathrm{d}x\, d_{e^\pm}^{\alpha 1 \rho}(x) = a_0^{\alpha\rho} + a_1^{\alpha\rho} \int_{1-\epsilon}^{1^-} \mathrm{d}x\, x^{a_2^{\alpha\rho}} (1-x)^{a_3^{\alpha\rho}} \tag{18}$$

Approximately

$$\int_{1-\epsilon}^{1^+} \mathrm{d}x\, d_{e^\pm}^{\alpha 1 \rho}(x) = a_0^{\alpha\rho} + a_1^{\alpha\rho} \int_{1-\epsilon}^{1^-} \mathrm{d}x\, (1-x)^{a_3^{\alpha\rho}} = a_0^{\alpha\rho} + a_1^{\alpha\rho} \int_{0^+}^{\epsilon} \mathrm{d}\xi\, \xi^{a_3^{\alpha\rho}} \tag{19}$$

and therefore

$$\int_{1-\epsilon}^{1^+} \mathrm{d}x\, d_{e^\pm}^{\alpha 1 \rho}(x) = a_0^{\alpha\rho} + a_1^{\alpha\rho} \frac{1-\epsilon^{a_3^{\alpha\rho}+1}}{a_3^{\alpha\rho}+1} \tag{20}$$

This simple approximation is good enough

74d ⟨*Update circe1 parameters* 33a⟩+≡

```
      circe1_params%elect0 = circe1_params%a1(0) + circe1_params%a1(1) * KIREPS**(circe1
      circe1_params%elect0 = circe1_params%elect0 / KIREPS
      circe1_params%gamma0 = circe1_params%a1(4) * KIREPS**(circe1_params%a1(5)+1) / (ci
      circe1_params%gamma0 = circe1_params%gamma0 / KIREPS
```

but we can also use incomplete Beta functions for the exact result:

75a ⟨*Alternative: Update circe1 parameters* 75a⟩≡
```
circe1_params%elect0 = circe1_params%a1(0) + circe1_params%a1(1) * beta (circe1_pa
                 * (1d0 - betinc (circe1_params%a1(2)+1, circe1_params%a1(3)+1, 1d0 -
circe1_params%elect0 = circe1_params%elect0 / KIREPS
circe1_params%gamma0 = circe1_params%a1(7) + circe1_params%a1(4) * beta (circe1_pa
                 * betinc (circe1_params%a1(5)+1, circe1_params%a1(6)+1, KIREPS)
circe1_params%gamma0 = circe1_params%gamma0 / KIREPS
```
Uses beta 105.

75b ⟨*Alternative: Local variables for* circes 75b⟩≡
```
real(kind=double) :: betinc
external betinc
```

75c ⟨*Calculate version 1 of the non-singular* e^+e^- *distribution* 75c⟩≡
```
if (x1 .gt. 1d0) then
   d1 = 0d0
elseif (x1 .ge. (1d0 - KIREPS)) then
   d1 = circe1_params%elect0
elseif (x1 .ge. 0d0) then
   d1 = circe1_params%a1(1) * x1**circe1_params%a1(2) * (1d0 - x1)**circe1_params%
else
   d1 = 0d0
endif
if (x2 .gt. 1d0) then
   d2 = 0d0
elseif (x2 .ge. (1d0 - KIREPS)) then
   d2 = circe1_params%elect0
elseif (x2 .ge. 0d0) then
   d2 = circe1_params%a1(1) * x2**circe1_params%a1(2) * (1d0 - x2)**circe1_params%
else
   d2 = 0d0
endif
kirkee = d1 * d2
```
Uses d1 16a, d2 16c, and kirkee 74b.

75d ⟨*Calculate version 1 of the non-singular* $e^\pm\gamma$ *distribution* 75d⟩≡
```
if (x1 .gt. 1d0) then
   d1 = 0d0
elseif (x1 .ge. (1d0 - KIREPS)) then
   d1 = circe1_params%elect0
elseif (x1 .ge. 0d0) then
   d1 = circe1_params%a1(1) * x1**circe1_params%a1(2) * (1d0 - x1)**circe1_params%
else
   d1 = 0d0
endif
if (x2 .gt. 1d0) then
   d2 = 0d0
elseif (x2 .gt. KIREPS) then
   d2 = circe1_params%a1(4) * x2**circe1_params%a1(5) * (1d0 - x2)**circe1_params%
elseif (x2 .ge. 0d0) then
```

```
        d2 = circe1_params%gamma0
     else
        d2 = 0d0
     endif
     kirkeg = d1 * d2
```
Uses `d1` 16a, `d2` 16c, and `kirkeg` 76c.

76a ⟨*Calculate version 1 of the non-singular $\gamma\gamma$ distribution* 76a⟩≡
```
     if (x1 .gt. 1d0) then
        d1 = 0d0
     elseif (x1 .gt. KIREPS) then
        d1 = circe1_params%a1(4) * x1**circe1_params%a1(5) * (1d0 - x1)**circe1_params%
     elseif (x1 .ge. 0d0) then
        d1 = circe1_params%gamma0
     else
        d1 = 0d0
     endif
     if (x2 .gt. 1d0) then
        d2 = 0d0
     elseif (x2 .gt. KIREPS) then
        d2 = circe1_params%a1(4) * x2**circe1_params%a1(5) * (1d0 - x2)**circe1_params%
     elseif (x2 .ge. 0d0) then
        d2 = circe1_params%gamma0
     else
        d2 = 0d0
     endif
     kirkgg = d1 * d2
```
Uses `d1` 16a, `d2` 16c, and `kirkgg` 77a.

76b ⟨*Public subroutines* 31a⟩+≡
```
  public :: kirkeg
```
Uses `kirkeg` 76c.

76c ⟨*Module subroutines* 31b⟩+≡
```
  function kirkeg (x1, x2)
    real(kind=double) :: x1, x2
    real(kind=double) :: kirkeg
    real(kind=double) :: d1, d2
    ⟨Initialization check 32g⟩
    kirkeg = -1.0
    if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
       ⟨Calculate version 1 of the non-singular e±γ distribution 75d⟩
    ⟨else handle invalid versions 37b⟩
  end function kirkeg
```

Defines:
`kirkeg`, used in chunks 73b, 75d, and 76b.
Uses `d1` 16a and `d2` 16c.

76d ⟨*Public subroutines* 31a⟩+≡
```
  public :: kirkgg
```
Uses `kirkgg` 77a.

77a ⟨*Module subroutines* 31b⟩+≡
```
function kirkgg (x1, x2)
  real(kind=double) :: x1, x2
  real(kind=double) :: kirkgg
  real(kind=double) :: d1, d2
  ⟨Initialization check 32g⟩
  kirkgg = -1.0
  if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
     ⟨Calculate version 1 of the non-singular γγ distribution 76a⟩
  ⟨else handle invalid versions 37b⟩
end function kirkgg
```

Defines:
`kirkgg`, used in chunks 73b and 76.
Uses `d1` 16a and `d2` 16c.

77b ⟨*Alternative: Subroutines* 77b⟩≡
```
function betinc (a, b, x)
  real(kind=double) :: x, a, b
  real(kind=double) :: betinc
  real(kind=double) :: bt
  if (x .lt. 0d0 .or. x .gt. 1d0) then
     betinc = 0d0
  else
    if (x .eq. 0d0 .or. x .eq. 1d0) then
       bt = 0d0
    else
       bt = exp(dlogam(a+b)-dlogam(a)-dlogam(b) &
                + a*log(x) + b*log(1d0-x))
    end if
    if (x .lt. (a+1d0)/ (a+b+2d0)) then
      betinc = bt*betacf (a, b, x) / a
    else
      betinc = 1d0 - bt*betacf (b, a, 1d0-x) / b
    end if
  end if
end function betinc
```

77c ⟨*Alternative: Subroutines* 77b⟩+≡
```
function betacf (a, b, x)
  real(kind=double) :: x, a, b
  real(kind=double) :: betacf
  integer, parameter :: itmax = 100
  real(kind=double), parameter = eps = 3d-7
  real(kind=double) :: am, bm, curr, prev, qab, qap, qam, bz, &
                       ap, bp, app, bpp, em, tem, d
  integer :: m
  am = 1d0
  bm = 1d0
  curr = 1d0
```

```
    qab = a + b
    qap = a + 1d0
    qam = a - 1d0
    bz = 1d0 - qab * x / qap
    do m = 1, ITMAX
      em = m
      tem = 2*em
      d = em * (b - m) * x / ((qam + tem) * (a + tem))
      ap = curr + d*am
      bp = bz + d*bm
      d = - (a + em) * (qab + em) * x / ((a + tem) * (qap + tem))
      app = ap + d * curr
      bpp = bp + d * bz
      prev = curr
      am = ap / bpp
      bm = bp / bpp
      curr = app / bpp
      bz = 1d0
      if (abs (curr - prev) .lt. EPS * abs (curr)) then
         betacf = curr
         return
      end if
    end do
    print *, 'betacf: failed to converge'
    betacf = 0d0
  end
```

6.5 Generators

6.5.1 Random-Number Generator

The generator routines do not fix or provide a random-number generator. The caller has to provide an implementation which is transferred to the subroutines in one of two possible forms:

1. as a subroutine which generates a single random number, working on an implicit external state
2. as an object with a method the generates a single random number, working on an internal state

These snippets should be used by the procedures that use a RNG:

78a ⟨*RNG dummy arguments* 78a⟩≡

```
rng, rng_obj
```

78b ⟨*RNG dummy declarations* 78b⟩≡

```
procedure(rng_proc), optional :: rng
class(rng_type), intent(inout), optional :: rng_obj
```

Uses `rng_proc` 79c and `rng_type` 79e.

79a ⟨*RNG: generate* `u` 79a⟩≡

```
  call rng_call (u, ⟨RNG dummy arguments 78a⟩)
```

Uses `rng_call` 79b.

79b ⟨*Module subroutines* 31b⟩+≡

```
    subroutine rng_call (u, ⟨RNG dummy arguments 78a⟩)
      real(kind=double), intent(out) :: u
      ⟨RNG dummy declarations 78b⟩
      if (present (rng)) then
         call rng (u)
      else if (present (rng_obj)) then
         call rng_obj%generate (u)
      else
         call circem ('PANIC', &
              'generator requires either rng or rng_obj argument')
      end if
    end subroutine rng_call
```

Defines:
`rng_call`, used in chunk 79a.
Uses `circem` 86e.

This defines the procedure version of the RNG, corresponding to the traditional F77 `external` interface. The abstract interface enables the compiler to check conformance.

79c ⟨*Abstract interfaces* 79c⟩≡

```
    abstract interface
      subroutine rng_proc (u)
        import :: double
        real(kind=double), intent(out) :: u
      end subroutine rng_proc
    end interface
```

Defines:
`rng_proc`, used in chunks 78b and 79c.

Here we define the object version of the RNG. It has to implement a `generate` method which parallels the `rng_proc` procedure above.

79d ⟨*Public types* 79d⟩≡

```
    public :: rng_type
```

Uses `rng_type` 79e.

79e ⟨*Abstract types* 79e⟩≡

```
    type, abstract :: rng_type
     contains
       procedure(rng_generate), deferred :: generate
    end type rng_type
```

Defines:
`rng_type`, used in chunks 78–80.
Uses `rng_generate` 80a.

80a ⟨*Abstract interfaces* 79c⟩+≡
```
abstract interface
   subroutine rng_generate (rng_obj, u)
     import :: rng_type, double
     class(rng_type), intent(inout) :: rng_obj
     real(kind=double), intent(out) :: u
   end subroutine rng_generate
end interface
```

Defines:
  `rng_generate`, used in chunk 79e.
Uses `rng_type` 79e.

6.5.2 Version 1

Beta distributions have the practical advantage that they have been popular among mathematicians.[?]

80b ⟨*Public subroutines* 31a⟩+≡
```
public :: girce
```
Uses `girce` 80c.

80c ⟨*Module subroutines* 31b⟩+≡
```
subroutine girce (x1, x2, p1, p2, ⟨RNG dummy arguments 78a⟩)
  real(kind=double), intent(out) :: x1, x2
  integer :: p1, p2
  ⟨RNG dummy declarations 78b⟩
  real(kind=double) :: u, w
⟨Initialization check 32g⟩
⟨x1m, x2m kludge, part 1 81b⟩
⟨Select particles p1 and p2 81a⟩
      if (abs(p1) .eq. C1_ELECTRON) then
         if (abs(p2) .eq. C1_ELECTRON) then
            call gircee (x1, x2, ⟨RNG dummy arguments 78a⟩)
         else if (p2 .eq. C1_PHOTON) then
            call girceg (x1, x2, ⟨RNG dummy arguments 78a⟩)
         end if
      else if (p1 .eq. C1_PHOTON) then
         if (abs(p2) .eq. C1_ELECTRON) then
            call girceg (x2, x1, ⟨RNG dummy arguments 78a⟩)
         else if (p2 .eq. C1_PHOTON) then
            call gircgg (x1, x2, ⟨RNG dummy arguments 78a⟩)
         end if
      end if
⟨x1m, x2m kludge, part 2 81c⟩
end subroutine girce
```

Defines:
  `girce`, used in chunks 80c, 20a, and 80.
Uses `C1_ELECTRON` 11b, `C1_PHOTON` 11b, `gircee` 81e, `girceg` 82c, and `gircgg` 83c.

81a ⟨*Select particles* p1 *and* p2 81a⟩≡
```
w = 1d0 / (1d0 + circgg (-1d0, -1d0))
⟨RNG: generate u 79a⟩
if (u*u .le. w) then
   p1 = C1_POSITRON
else
   p1 = C1_PHOTON
end if
⟨RNG: generate u 79a⟩
if (u*u .le. w) then
   p2 = C1_ELECTRON
else
   p2 = C1_PHOTON
end if
```
Uses `C1_ELECTRON` 11b, `C1_PHOTON` 11b, `C1_POSITRON` 11b, and `circgg` 43c.

The flavor selection is incorrect, because the relative weights depend on the minimum energy fractions. We resort to a moderately inefficient kludge, because we don't have the distribution functions available yet. We'll have to implement incomplete Beta functions and other horrible things for this. Fortunately, the efficiency can not drop below the relative contribution of e^+e^-.

81b ⟨x1m, x2m *kludge, part 1* 81b⟩≡
```
do
```
Crude rejection:

81c ⟨x1m, x2m *kludge, part 2* 81c⟩≡
```
   if ((x1 .ge. circe1_params%x1m) .and. (x2 .ge. circe1_params%x2m)) exit
end do
```

81d ⟨*Public subroutines* 31a⟩+≡
```
public :: gircee
```
Uses `gircee` 81e.

81e ⟨*Module subroutines* 31b⟩+≡
```
subroutine gircee (x1, x2, ⟨RNG dummy arguments 78a⟩)
  real(kind=double), intent(out) :: x1, x2
  ⟨RNG dummy declarations 78b⟩
  real(kind=double) :: u
  ⟨Initialization check 32g⟩
  x1 = 1
  x2 = 1
  if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
     ⟨Generate version 1 of the e+e- distribution 82a⟩
  ⟨else handle invalid versions 37b⟩
end subroutine gircee
```

Defines:
`gircee`, used in chunks 20, 81e, 21d, 80c, and 81d.

For version 1 of the parametrizations we rely on `girceb`, a fast generator of β-distribitions:

$$\beta^{a,b}_{x_{\min},x_{\max}}(x) \quad = \quad x^{a-1}(1-x)^{b-1}\cdot\frac{\Theta(x_{\max}-x)\Theta(x-x_{\min})}{I(x_{\min},a,b)-I(x_{\max},a,b)} \qquad (21)$$

$$I(x,a,b) \quad = \quad \int_x^1 d\xi\, \xi^{a-1}(1-\xi)^{b-1} \tag{22}$$

82a ⟨*Generate version 1 of the* e^+e^- *distribution* 82a⟩≡
```
        ⟨RNG: generate u 79a⟩
        if (u .le. circe1_params%a1(0)) then
           x1 = 1d0
        else
           x1 = 1d0 - girceb (0d0, 1d0-circe1_params%x1m, &
                              circe1_params%a1(3)+1d0, circe1_params%a1(2)+1d0, &
                              ⟨RNG dummy arguments 78a⟩)
        endif
        ⟨RNG: generate u 79a⟩
        if (u .le. circe1_params%a1(0)) then
           x2 = 1d0
        else
           x2 = 1d0 - girceb (0d0, 1d0-circe1_params%x2m, &
                              circe1_params%a1(3)+1d0, circe1_params%a1(2)+1d0, &
                              ⟨RNG dummy arguments 78a⟩)
        endif
```
Uses `girceb` 84b.

82b ⟨*Public subroutines* 31a⟩+≡
```
    public :: girceg
```
Uses `girceg` 82c.

82c ⟨*Module subroutines* 31b⟩+≡
```
    subroutine girceg (x1, x2, ⟨RNG dummy arguments 78a⟩)
      real(kind=double), intent(out) :: x1, x2
      ⟨RNG dummy declarations 78b⟩
      real(kind=double) :: u
      ⟨Initialization check 32g⟩
      x1 = 1
      x2 = 1
      if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
         ⟨Generate version 1 of the e±γ distribution 82d⟩
      ⟨else handle invalid versions 37b⟩
    end subroutine girceg
```

Defines:
`girceg`, used in chunks 20c, 80c, and 82b.

82d ⟨*Generate version 1 of the* $e^\pm\gamma$ *distribution* 82d⟩≡
```
        ⟨RNG: generate u 79a⟩
        if (u .le. circe1_params%a1(0)) then
           x1 = 1d0
        else
           x1 = 1d0 - girceb (0d0, 1d0-circe1_params%x1m, &
                              circe1_params%a1(3)+1d0, circe1_params%a1(2)+1d0, &
                              ⟨RNG dummy arguments 78a⟩)
        endif
        x2 = girceb (circe1_params%x2m, 1d0, &
```

```
                          circe1_params%a1(5)+1d0, circe1_params%a1(6)+1d0, &
                          ⟨RNG dummy arguments 78a⟩)
```

Uses `girceb` 84b.

83a ⟨*Public subroutines* 31a⟩+≡

```
  public :: gircgg
```

Uses `gircgg` 83c.

83b ⟨*Module subroutines* 31b⟩+≡

```
  subroutine gircgg (x1, x2, ⟨RNG dummy arguments 78a⟩)
    real(kind=double), intent(out) :: x1, x2
    ⟨RNG dummy declarations 78b⟩
    ⟨Initialization check 32g⟩
    x1 = 1
    x2 = 1
    if ((circe1_params%ver .eq. 1) .or. (circe1_params%ver .eq. 0)) then
       ⟨Generate version 1 of the γγ distribution 83c⟩
    ⟨else handle invalid versions 37b⟩
  end subroutine gircgg
```

Uses `gircgg` 83c.

83c ⟨*Generate version 1 of the* $\gamma\gamma$ *distribution* 83c⟩≡

```
       x1 = girceb (circe1_params%x1m, 1d0, &
                    circe1_params%a1(5)+1d0, circe1_params%a1(6)+1d0, &
                    ⟨RNG dummy arguments 78a⟩)
       x2 = girceb (circe1_params%x2m, 1d0, &
                    circe1_params%a1(5)+1d0, circe1_params%a1(6)+1d0, &
                    ⟨RNG dummy arguments 78a⟩)
```

Defines:
`gircgg`, used in chunks 20c, 80c, and 83.
Uses `girceb` 84b.

6.5.3 Version 2

Retired.

6.5.4 Version 3 and 4

Identical to version 1.

6.6 Utilities

For version 1 of the parametrizations we need a fast generator of β-distributions:

$$\beta^{a,b}_{x_{\min},x_{\max}}(x) = x^{a-1}(1-x)^{b-1} \cdot \frac{\Theta(x_{\max}-x)\Theta(x-x_{\min})}{I(x_{\min},a,b)-I(x_{\max},a,b)} \tag{23}$$

with the *incomplete Beta-function* I*:*

$$I(x,a,b) = \int_x^1 d\xi\, \xi^{a-1}(1-\xi)^{b-1} \tag{24}$$

$$B(a,b) = I(0,a,b) \tag{25}$$

This problem has been studied extensively [?] and we can use an algorithm [18] that is very fast for $0 < a \leq 1 \leq b$, which turns out to be the case in our application.

84a ⟨*Public subroutines* 31a⟩+≡
```
public :: girceb
```
Uses `girceb` 84b.

84b ⟨*Module subroutines* 31b⟩+≡
```
function girceb (xmin, xmax, a, b, ⟨RNG dummy arguments 78a⟩)
  real(kind=double) :: xmin, xmax, a, b
  real(kind=double) :: girceb
  ⟨RNG dummy declarations 78b⟩
  real(kind=double) :: t, p, u, umin, umax, x, w
  ⟨Check a and b 84c⟩
  ⟨Set up girceb parameters 84d⟩
  do
     ⟨Generate a trial x and calculate its weight w 85a⟩
     ⟨RNG: generate u 79a⟩
     if (w .gt. u) exit
  end do
  girceb = x
end function girceb
```

Defines:
`girceb`, used in chunks 84b, 82–84, and 86b.

In fact, this algorithm works for $0 < a \leq 1 \leq b$ only:

84c ⟨*Check* `a` *and* `b` 84c⟩≡
```
if ((a .ge. 1d0) .or. (b .le. 1d0)) then
   girceb = -1d0
   call circem ('ERROR', 'beta-distribution expects a<1<b')
   return
end if
```
Uses `circem` 86e and `girceb` 84b.

The trick is to split the interval $[0, 1]$ into two parts $[0, t]$ and $[t, 1]$. In these intervals we obviously have

$$x^{a-1}(1-x)^{b-1} \leq \begin{cases} x^{a-1} & \text{for } x \leq t \\ t^{a-1}(1-x)^{b-1} & \text{for } x \geq t \end{cases} \tag{26}$$

because we have assumed that $0 < a < 1 < b$. The integrals of the two dominating distributions are t^a/a and $t^{a-1}(1-t)^b/b$ respectively and therefore the probability for picking a random number from the first interval is

$$P(x \leq t) = \frac{bt}{bt + a(1-t)^b} \tag{27}$$

We postpone the discussion of the choice of t until later:

84d ⟨*Set up* `girceb` *parameters* 84d⟩≡
```
⟨Set up best value for t 86c⟩
p = b*t / (b*t + a * (1d0 - t)**b)
```

The dominating distributions can be generated by simple mappings

$$\phi : [0,1] \to [0,1] \tag{28}$$

$$u \mapsto \begin{cases} t\left(\frac{u}{p}\right)^{\frac{1}{a}} & < t \text{ for } u < p \\ t & = t \text{ for } u = p \\ 1-(1-t)\left(\frac{1-u}{1-p}\right)^{\frac{1}{b}} & > t \text{ for } u > p \end{cases} \tag{29}$$

The beauty of the algorithm is that we can use a single uniform deviate u for both intervals:

85a ⟨*Generate a trial* `x` *and calculate its weight* `w` 85a⟩≡

```
⟨RNG: generate u 79a⟩
u = umin + (umax - umin) * u
if (u .le. p) then
   x = t * (u/p)**(1d0/a)
   w = (1d0 - x)**(b-1d0)
else
   x = 1d0 - (1d0 - t) * ((1d0 - u)/(1d0 - p))**(1d0/b)
   w = (x/t)**(a-1d0)
end if
```

The weights that are derived by dividing the distribution by the dominating distributions are already normalized correctly:

$$w : [0,1] \to [0,1] \tag{30}$$

$$x \mapsto \begin{cases} (1-x)^{b-1} & \in [(1-t)^{b-1}, 1] \text{ for } x \le t \\ \left(\frac{x}{t}\right)^{a-1} & \in [t^{1-a}, 1] \text{ for } x \ge t \end{cases} \tag{31}$$

To derive $u_{\min,\max}$ from $x_{\min,\max}$ we can use ϕ^{-1}:

$$\phi^{-1} : [0,1] \to [0,1] \tag{32}$$

$$x \mapsto \begin{cases} p\left(\frac{x}{t}\right)^{a} & < p \text{ for } x < t \\ p & = p \text{ for } x = t \\ 1-(1-p)\left(\frac{1-x}{1-t}\right)^{b} & > p \text{ for } x > t \end{cases} \tag{33}$$

We start with $u_{\min}$. For efficiency, we handle the most common cases (small $x_{\min}$) first:

85b ⟨*Set up* `girceb` *parameters* 84d⟩+≡

```
if (xmin .le. 0d0) then
   umin = 0d0
elseif (xmin .lt. t) then
   umin = p * (xmin/t)**a
elseif (xmin .eq. t) then
   umin = p
elseif (xmin .lt. 1d0) then
   umin = 1d0 - (1d0 - p) * ((1d0 - xmin)/(1d0 - t))**b
else
   umin = 1d0
endif
```

Same procedure for $u_{\max}$; again, handle the most common cases (large $x_{\max}$) first:

86a ⟨*Set up* `girceb` *parameters* 84d⟩+≡

```
        if (xmax .ge. 1d0) then
           umax = 1d0
        elseif (xmax .gt. t) then
           umax = 1d0 - (1d0 - p) * ((1d0 - xmax)/(1d0 - t))**b
        elseif (xmax .eq. t) then
           umax = p
        elseif (xmax .gt. 0d0) then
           umax = p * (xmax/t)**a
        else
           umax = 0d0
        endif
```

Check for absurd cases.

86b ⟨*Set up* `girceb` *parameters* 84d⟩+≡

```
        if (umax .lt. umin) then
           girceb = -1d0
           return
        endif
```

Uses `girceb` 84b.

It remains to choose he best value for t. The rejection efficiency ϵ of the algorithm is given by the ratio of the dominating distribution and the distribution

$$\frac{1}{\epsilon(t)} = \frac{B(a,b)}{ab}\left(bt^a + at^{a-1}(1-t)^b\right). \tag{34}$$

It is maximized for

$$bt - bt(1-t)^{b-1} + (a-1)(1-t)^b = 0 \tag{35}$$

This equation has a solution which can be determined numerically. While this determination is far too expensive compared to a moderate loss in efficiency, we could perform it once after fitting the coefficients a, b. Nevertheless, it has been shown,[18] that

$$t = \frac{1-a}{b+1-a} \tag{36}$$

results in non-vanishing efficiency for all values $1 < a \leq 1 \leq b$. Empirically we have found efficiencies of at least 80% for this choice, which is enough for our needs.

86c ⟨*Set up best value for t* 86c⟩≡

```
        t = (1d0 - a) / (b + 1d0 - a)
```

86d ⟨*Public subroutines* 31a⟩+≡

```
    public :: circem
```

Uses `circem` 86e.

86e ⟨*Module subroutines* 31b⟩+≡

```
    subroutine circem (errlvl, errmsg)
      character(len=*) :: errlvl, errmsg
      integer, save :: errcnt = 0
```

```
      if (errlvl .eq. 'MESSAGE') then
         print *, 'circe1:message: ', errmsg
      else if (errlvl .eq. 'WARNING') then
         if (errcnt .lt. 100) then
            errcnt = errcnt + 1
            print *, 'circe1:warning: ', errmsg
         else if (errcnt .eq. 100) then
            errcnt = errcnt + 1
            print *, 'circe1:message: more than 100 messages'
            print *, 'circe1:message: turning warnings off'
         end if
      else if (errlvl .eq. 'ERROR') then
         if (errcnt .lt. 200) then
            errcnt = errcnt + 1
            print *, 'circe1:error:   ', errmsg
         else if (errcnt .eq. 200) then
            errcnt = errcnt + 1
            print *, 'circe1:message: more than 200 messages'
            print *, 'circe1:message: turning error messages off'
         endif
      else if (errlvl .eq. 'PANIC') then
         if (errcnt .lt. 300) then
            errcnt = errcnt + 1
            print *, 'circe1:panic:   ', errmsg
         else if (errcnt .eq. 300) then
            errcnt = errcnt + 1
            print *, 'circe1:message: more than 300 messages'
            print *, 'circe1:message: turning panic messages off'
         end if
      else
         print *, 'circe1:panic:    invalid error code ', errlvl
      end if
    end subroutine circem
```

Defines:
`circem`, used in chunks 32–34, 36–39, 50, 51a, 53–57, 61, 63, 65a, 68, 69, 79b, 84c, and 86d.

6.7 Examples

6.7.1 Distributions

87 ⟨circe1_plot.f90 87⟩≡

```
  program circe1_plot
    use kinds
    use circe1

    implicit none

    real(kind=double) :: xmin, xmax, y, roots
```

```
    integer :: xory, nstep, p1, p2, acc, ver, rev, i
    real(kind=double) :: x, logx, d
    read *, xory, xmin, xmax, nstep, y, p1, p2, roots, acc, ver, rev
    call circes (0d0, 0d0, roots, acc, ver, rev, 0)
    do i = 0, nstep
       logx = log (xmin) + i * log (xmax/xmin) / nstep
       x = exp (logx)
       d = 0d0
       if (xory .eq. 1) then
          if (p1 .eq. C1_PHOTON) then
             d = circe (x, y, p1, p2)
          else
             d = circe (1d0 - x, y, p1, p2)
          end if
       else if (xory .eq. 2) then
          if (p1 .eq. C1_PHOTON) then
             d = circe (y, x, p1, p2)
          else
             d = circe (y, 1d0 - x, p1, p2)
          end if
       end if
       if (d .gt. 1d-4) print *, x, d
    end do
  end program circe1_plot
```

Uses `C1_PHOTON` 11b, `circe` 31b, and `circes` 32a.

6.7.2 Library functions

If Fortran77 only had first class functions, then the following cruft would not be necessary. OK, here's the outline of the adaptive Gauss integration routine from CERNLIB:

88 ⟨*Part one of Gaussian integration* 88⟩≡

```
        real(kind=double) :: f, a, b, eps
        external f
        real(kind=double), parameter :: Z1 = 1, HF = Z1/2, CST = 5*Z1/1000
        integer :: i
        real(kind=double) :: h, const, aa, bb, c1, c2, s8, s16, u
        ⟨Gaussian weights 91a⟩
        h = 0
        if (b .eq. a) go to 99
        const = CST / dabs(b-a)
        bb = a
      1 continue
           aa = bb
           bb = b
      2 continue
           c1 = HF*(bb+aa)
           c2 = HF*(bb-aa)
           s8 = 0
```

```
            do i = 1, 4
               u = c2*x(i)
```

Here are now the first two function calls that we have to fill in later in various ways:

89a ⟨*Function call stub* 89a⟩≡

```
               s8 = s8 + w(i) * (f (c1+u) + f (c1-u))
```

Continuing

89b ⟨*Part two of Gaussian integration* 89b⟩≡

```
            end do
            s16 = 0
            do i = 5, 12
               u = c2*x(i)
```

And here are the other two function calls:

89c ⟨*Function call stub* 89a⟩+≡

```
               s16 = s16 + w(i) * (f (c1+u) + f (c1-u))
```

Terminating:

89d ⟨*Part three of Gaussian integration* 89d⟩≡

```
            end do
            s16 = c2*s16
         if (dabs(s16-c2*s8) .le. eps*(1+dabs(s16))) then
            h = h + s16
            if (bb .ne. b) go to 1
         else
            bb = c1
            if (1 + const*dabs(c2) .ne. 1) go to 2
            h = 0
            print *, 'gauss: too high accuracy required'
            go to 99
         end if
      99 continue
```

This one is still reasonably straightforward

$$\mathtt{gauss1} : (f, a, b) \mapsto \int_a^b dx\, f(x) \tag{37}$$

89e ⟨circe1_sample.f90: public 15a⟩+≡

```
    public :: gauss1
```

Uses gauss1 89f.

89f ⟨circe1_sample.f90: subroutines 15b⟩+≡

```
    function gauss1 (f, a, b, eps)
      real(kind=double) :: gauss1
  ⟨Part one of Gaussian integration 88⟩
      s8 = s8 + w(i) * (f (c1+u) + f (c1-u))
  ⟨Part two of Gaussian integration 89b⟩
      s16 = s16 + w(i) * (f (c1+u) + f (c1-u))
      ⟨Part three of Gaussian integration 89d⟩
      gauss1 = h
    end function gauss1
```

Defines:
gauss1, used in chunks 89f, 15c, and 89.

But this almost identical repeat

$$\texttt{gaussx} : (f, a, b) \mapsto \left(y \mapsto \int_a^b dx\, f(y, x) \right) \tag{38}$$

would not be necassary in a modern programming language with currying:

90a ⟨circe1_sample.f90: public 15a⟩+≡
```
public :: gaussx
```
Uses gaussx 90b.

90b ⟨circe1_sample.f90: subroutines 15b⟩+≡
```
    function gaussx (f, y, a, b, eps)
      real(kind=double) :: y
      real(kind=double) :: gaussx
  ⟨Part one of Gaussian integration 88⟩
      s8 = s8 + w(i) * (f (y, c1+u) + f (y, c1-u))
  ⟨Part two of Gaussian integration 89b⟩
      s16 = s16 + w(i) * (f (y, c1+u) + f (y, c1-u))
      ⟨Part three of Gaussian integration 89d⟩
      gaussx = h
    end function gaussx
```

Defines:
gaussx, used in chunk 90.

Fortunately, this is the last one we need

$$\begin{aligned}\texttt{gauss2} : (f, a, b, a_1, b_1) \mapsto \int_a^b dx \int_{a_1}^{b_1} dy\, f(x, y) \\ = \texttt{gauss1}\,(\texttt{gaussx}(f, a, b), a_1, b_1)\end{aligned} \tag{39}$$

90c ⟨circe1_sample.f90: public 15a⟩+≡
```
public :: gauss2
```
Uses gauss2 90d.

90d ⟨circe1_sample.f90: subroutines 15b⟩+≡
```
  function gauss2 (f, a, b, a1, b1, eps)
    real(kind=double) :: a1, b1
    real(kind=double) :: gauss2
  ⟨Part one of Gaussian integration 88⟩
    s8 = s8 + w(i) * (gaussx (f, c1+u, a1, b1, eps) &
                    + gaussx (f, c1-u, a1, b1, eps))
  ⟨Part two of Gaussian integration 89b⟩
    s16 = s16 + w(i) * (gaussx (f, c1+u, a1, b1, eps) &
                      + gaussx (f, c1-u, a1, b1, eps))
  ⟨Part three of Gaussian integration 89d⟩
    gauss2 = h
  end function gauss2
```

Defines:
gauss2, used in chunks 90d, 15c, 16e, and 90.
Uses gaussx 90b.

91a ⟨*Gaussian weights* 91a⟩≡
```
      real(kind=double), dimension(12), parameter :: &
        x = (/ 9.6028985649753623d-1, &
               7.9666647741362674d-1, &
               5.2553240991632899d-1, &
               1.8343464249564980d-1, &
               9.8940093499164993d-1, &
               9.4457502307323258d-1, &
               8.6563120238783174d-1, &
               7.5540440835500303d-1, &
               6.1787624440264375d-1, &
               4.5801677765722739d-1, &
               2.8160355077925891d-1, &
               9.5012509837637440d-2 /), &
        w = (/ 1.0122853629037626d-1, &
               2.2238103445337447d-1, &
               3.1370664587788729d-1, &
               3.6268378337836198d-1, &
               2.7152459411754095d-2, &
               6.2253523938647893d-2, &
               9.5158511682492785d-2, &
               1.2462897125553387d-1, &
               1.4959598881657673d-1, &
               1.6915651939500254d-1, &
               1.8260341504492359d-1, &
               1.8945061045506850d-1 /)
```

6.7.3 Generators

6.8 Dumping Parameters

91b ⟨params.f90 91b⟩≡
```
    program params
      use kinds
      use circe1

      implicit none
      integer :: acc, ver, i
      real(kind=double), dimension(7), parameter :: roots = &
        (/ 90D0, 170D0, 350D0, 500D0, 800D0, 1000D0, 1500D0 /)
        do ver = 7, 8
          print *, "VERSION ", ver
           do acc = TESLA, XBNDEE
              do 12 i = 1, 7
                 print *, "=============================="
```

```
                call circes (0d0, 0d0, roots(i), acc, ver, 20020307, 0)
                call dump ()
             end do
          end do
       end do
    end program params
```

Uses TESLA 13a, XBNDEE 13a, and circes 32a.

92 ⟨params.f90 91b⟩+≡

```
    subroutine dump
      ⟨Accelerator codes 13a⟩
      character(len=9) :: name
      select case (acc)
        case (SBAND)
           name = 'SBAND'
        case (TESLA)
           name = 'TESLA'
        case (JLCNLC)
           name = 'JLCNLC'
        case (SBNDEE)
           name = 'SBAND/EE'
        case (TESLEE)
           name = 'TESLA/EE'
        case (XBNDEE)
           name = 'JLCNLC/EE'
        case (ILC)
           name = 'ILC'
        case default
           print *, "Accelerator mode not recognized"
      end select
      write (*, 1000) name, circe1_params%roots
      write (*, 1001) 'e^+/e^-', circe1_params%lumi
      write (*, 1002) 'e^+/e^-', circe1_params%a1(0)
      write (*, 1003) 'e^+/e^-', 1 - circe1_params%a1(0)
      write (*, 1004) 'e^+/e^-', circe1_params%a1(1), circe1_params%a
      write (*, 1003) 'gamma', circe1_params%a1(7)
      write (*, 1004) 'gamma', circe1_params%a1(4), circe1_params%a1(5), circe1_params%a1(
   1000 format (A9, ' @ ', F5.0, ' GeV')
   1001 format (4X, A7, ' lumi             = ', F7.2,' * 10^32 cm^-2 sec^-1')
   1002 format (4X, A7, ' delta strength   = ', F9.5)
   1003 format (4X, A7, ' integral(cont.) = ', F9.5)
   1004 format (4X, A7, ' distribution     = ', F9.5, ' * x^{', F9.5, '} * (1-x)^{', F9.5,
    end subroutine dump
```

Uses ILC 13a, JLCNLC 13a, SBAND 13a, SBNDEE 13a, TESLA 13a, TESLEE 13a, and XBNDEE 13a.

7 Fitting

7.1 Version 1: Factorized Beta Distributions

93a ⟨*Copyleft notice* 29b⟩+≡
```
!
! Copyright (C) 1999-2015 by
!     Wolfgang Kilian <kilian@physik.uni-siegen.de>
!     Thorsten Ohl <ohl@physik.uni-wuerzburg.de>
!     Juergen Reuter <juergen.reuter@desy.de>
!     Christian Speckner <cnspeckn@googlemail.com>
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! Foundation, Inc., 675 Mass Ave, Cambridge, MA 02139, USA.
!
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
! This file has been stripped of most comments.  For documentation, refer
! to the source 'minuit.nw'
```

93b ⟨circe1_fit.f90 93b⟩≡
```
! circe1_fit.f90 -- fitting for circe
⟨Copyleft notice 29b⟩

module fit_routines
  use kinds

  implicit none
  private

⟨circe1_fit.f90: public 95a⟩

contains
⟨circe1_fit.f90: subroutines 95b⟩
end module fit_routines

program fit
  use kinds
  use fit_routines

  implicit none

  integer :: i, rcode
```

```
⟨Declare NPARAM 94a⟩
⟨Declare parameters 94b⟩
⟨Declare arguments 94c⟩

      ⟨Initialize parameters for circe1_fit.f90 94e⟩
      call mninit (5, 6, 7)
      ⟨Load parameters 94d⟩
      call mnseti ('CIRCE: fit version 1     ')
      argv(1) = 1
      call mnexcm (fct, 'SET PRINTOUT        ', argv, 1, rcode, 0d0)
      argv(1) = 1
      call mnexcm (fct, 'CALL FCT            ', argv, 1, rcode, 0d0)
      call mnexcm (fct, 'MIGRAD              ', argv, 0, rcode, 0d0)
      call mnexcm (fct, 'MINOS               ', argv, 0, rcode, 0d0)
      argv(1) = 3
      call mnexcm (fct, 'CALL FCT            ', argv, 1, rcode, 0d0)
      call mnexcm (fct, 'STOP                ', argv, 0, rcode, 0d0)

end program fit
```

Defines:
fit, used in chunks 94d, 111, and 115c.
Uses circe 31b and fct 95b 111c.

94a ⟨*Declare* NPARAM 94a⟩≡

```
    integer, parameter :: NPARAM = 6
```

Defines:
NPARAM, used in chunks 94 and 101b.

94b ⟨*Declare parameters* 94b⟩≡

```
  integer, dimension(NPARAM) :: pnum
  character(len=10), dimension(NPARAM) :: pname
  real(kind=double), dimension(NPARAM) :: pstart, pstep
```

Uses NPARAM 94a.

94c ⟨*Declare arguments* 94c⟩≡

```
  integer, parameter :: ARGC = 10
  real(kind=double), dimension(ARGC) :: argv
```

94d ⟨*Load parameters* 94d⟩≡

```
  do i = 1, NPARAM
     call mnparm (pnum(i), pname(i), pstart(i), pstep (i), 0d0, 0d0, rcode)
     if (rcode .ne. 0) then
        print *, "fit: MINUIT won''t accept parameter ", pnum(i)
        stop
     endif
  end do
```

Uses NPARAM 94a and fit 93b.

94e ⟨*Initialize parameters for* circe1_fit.f90 94e⟩≡

```
  data pnum   /     1,     2,       3,     4,     5,       6 /
  data pname  / '1_e', 'x_e', '1-x_e', '1_g', 'x_g', '1-x_g' /
  data pstart / -1.00, 20.00,    0.20, -1.00,  0.20,   20.00 /
  data pstep  /  0.01,  0.01,    0.01,  0.01,  0.01,    0.01 /
```

95a ⟨circe1_fit.f90: public 95a⟩≡
```
    public :: fct
```
Uses fct 95b 111c.

95b ⟨circe1_fit.f90: subroutines 95b⟩≡
```
    subroutine fct (nx, df, f, a, mode, g)
      integer :: nx, mode
      real(kind=double) :: f, g
      real(kind=double), dimension(:) :: df, a
      ⟨Local variables for fct (v1) 95e⟩
      if (mode .eq. 1) then
        ⟨Read input data (v1) 95c⟩
      else if (mode .eq. 2) then
        ⟨Calculate ∇f 99a⟩
      end if
      ⟨Calculate f (v1) 99b⟩
      end if
      if (mode .eq. 3) then
          ⟨Write output (v1) 101a⟩
      end if
    end subroutine fct
```

Defines:
fct, used in chunks 93b, 95a, 110, 111, and 115f.

95c ⟨*Read input data (v1)* 95c⟩≡
```
        ⟨Read data from file 95d⟩
        ⟨Fixup errors 97a⟩
        ⟨Normalize 97d⟩
```

95d ⟨*Read data from file* 95d⟩≡
```
      call gethst ('ee', NDATA, xee, fee, dfee, see, tee, pwr)
      call gethst ('eg', NDATA, xeg, feg, dfeg, seg, teg, pwr)
      call gethst ('ge', NDATA, xge, fge, dfge, sge, tge, pwr)
      call gethst ('gg', NDATA, xgg, fgg, dfgg, sgg, tgg, pwr)
```
Uses gethst 96a.

95e ⟨*Local variables for* fct *(v1)* 95e⟩≡
```
      integer, parameter :: NDATA = 20
      real(kind=double) :: see, tee, dtee
      real(kind=double) :: seg, teg, dteg
      real(kind=double) :: sge, tge, dtge
      real(kind=double) :: sgg, tgg, dtgg
      real(kind=double), dimension(2,0:NDATA+1,0:NDATA+1) :: xee, xeg, &
           xge, xgg
      real(kind=double), dimension(0:NDATA+1,0:NDATA+1) :: fee, dfee, &
           feg, dfeg, fge, dfge, fgg, dfgg
      real(kind=double) :: pwr
```

95f ⟨circe1_fit.f90: public 95a⟩+≡
```
    public :: gethst
```
Uses gethst 96a.

96a ⟨circe1_fit.f90: subroutines 95b⟩+≡

```
subroutine gethst (tag, ndata, x, f, df, s, t, pwr)
  character(len=2) :: tag
  integer :: ndata
  real(kind=double) :: s, t, pwr
  real(kind=double), dimension(2,0:ndata+1,0:ndata+1) :: x
  real(kind=double), dimension(0:ndata+1,0:ndata+1) :: f, df
  integer :: i, j
  open (10, file = 'lumidiff-'//tag//'.dat')
  read (10, *) pwr
  s = 0d0
  ⟨Read continuum, summing in s 96b⟩
  t = s
  ⟨Read single δ, summing in t 96c⟩
  ⟨Read double δ, summing in t 96e⟩
  close (10)
end subroutine gethst
```

Defines:
gethst, used in chunk 95.

96b ⟨*Read continuum, summing in* s 96b⟩≡

```
  do i = 1, ndata
     do j = 1, ndata
        read (10, *) x(1,i,j), x(2,i,j), f(i,j), df(i,j)
        s = s + f(i,j)
     end do
  end do
```

96c ⟨*Read single δ, summing in* t 96c⟩≡

```
  do i = 1, ndata
     read (10, *) x(1,i,0), f(i,0), df(i,0), &
                            f(i,ndata+1), df(i,ndata+1)
     x(1,i,ndata+1) = x(1,i,0)
     t = t + f(i,0) + f(i,ndata+1)
  end do
```

96d ⟨*Read single δ, summing in* t 96c⟩+≡

```
  do i = 1, ndata
     read (10, *) x(2,0,i), f(0,i), df(0,i), &
                            f(ndata+1,i), df(ndata+1,i)
     x(2,ndata+1,i) = x(2,0,i)
     t = t + f(0,i) + f(ndata+1,i)
  end do
```

96e ⟨*Read double δ, summing in* t 96e⟩≡

```
  read (10, *) f(0,0), df(0,0), f(0,ndata+1), df(0,ndata+1)
  t = t + f(0,0) + f(0,ndata+1)
  read (10, *) f(ndata+1,0), df(ndata+1,0), &
               f(ndata+1,ndata+1), df(ndata+1,ndata+1)
  t = t + f(ndata+1,0) + f(ndata+1,ndata+1)
```

Guinea-Pig does not provide the full error. A Monte Carlo study shows that it is a reasonable approximation to rescale the bin error by suitable factors. These factors are different for eahc distribution and the factors for the δ-pieces are bigger than those for the continuum parts. The follows factors are for the slow parameter set.

97a ⟨*Fixup errors* 97a⟩≡
```
      call fixerr (NDATA, dfee, 20d0, 30d0, 40d0)
      call fixerr (NDATA, dfeg, 15d0, 20d0,  0d0)
      call fixerr (NDATA, dfge, 15d0, 20d0,  0d0)
      call fixerr (NDATA, dfgg, 10d0,  0d0,  0d0)
```
Uses fixerr 97c.

97b ⟨circe1_fit.f90: public 95a⟩+≡
```
    public :: fixerr
```
Uses fixerr 97c.

97c ⟨circe1_fit.f90: subroutines 95b⟩+≡
```
    subroutine fixerr (ndata, df, c, sd, dd)
      integer :: ndata
      real(kind=double) :: c, sd, dd
      real(kind=double), dimension(0:ndata+1,0:ndata+1) :: df
      integer :: i, j
      do i = 1, NDATA
         do j = 1, NDATA
            df(i,j) = c * df(i,j)
         end do
      end do
      do i = 1, NDATA
         df(0,i) = sd * df(0,i)
         df(i,0) = sd * df(i,0)
         df(ndata+1,i) = sd * df(ndata+1,i)
         df(i,ndata+1) = sd * df(i,ndata+1)
      end do
      df(0,0) = dd * df(0,0)
      df(ndata+1,0) = dd * df(ndata+1,0)
      df(0,ndata+1) = dd * df(0,ndata+1)
      df(ndata+1,ndata+1) = dd * df(ndata+1,ndata+1)
    end subroutine fixerr
```

Defines:
fixerr, used in chunk 97.

The error on the integrated luminosity is obtained from adding the error in channels in quadrature.

97d ⟨*Normalize* 97d⟩≡
```
      dtee = sumsqu (NDATA, dfee)
      dteg = sumsqu (NDATA, dfeg)
      dtge = sumsqu (NDATA, dfge)
      dtgg = sumsqu (NDATA, dfgg)
```
Uses sumsqu 98b.

98a ⟨circe1_fit.f90: public 95a⟩+≡
```
    public :: sumsqu
```
Uses sumsqu 98b.

98b ⟨circe1_fit.f90: subroutines 95b⟩+≡
```
    function sumsqu (ndata, f)
      integer :: ndata
      real(kind=double) :: sumsqu
      real(kind=double), dimension(0:ndata+1,0:ndata+1) :: f
      integer :: i, j
      real(kind=double) :: s2
      s2 = 0
      do i = 0, NDATA+1
         do j = 0, NDATA+1
            s2 = s2 + f(i,j)*f(i,j)
         end do
      end do
      sumsqu = sqrt (s2)
    end function sumsqu
```
Defines:
sumsqu, used in chunks 97d and 98a.

98c ⟨*Normalize* 97d⟩+≡
```
      call scale (NDATA, 1d0/tee, fee)
      call scale (NDATA, 1d0/tee, dfee)
      call scale (NDATA, 1d0/tee, feg)
      call scale (NDATA, 1d0/tee, dfeg)
      call scale (NDATA, 1d0/tee, fge)
      call scale (NDATA, 1d0/tee, dfge)
      call scale (NDATA, 1d0/tee, fgg)
      call scale (NDATA, 1d0/tee, dfgg)
```
Uses scale 98e.

98d ⟨circe1_fit.f90: public 95a⟩+≡
```
    public :: scale
```
Uses scale 98e.

98e ⟨circe1_fit.f90: subroutines 95b⟩+≡
```
    subroutine scale (ndata, s, f)
      integer :: ndata
      real(kind=double) :: s
      real(kind=double), dimension(0:ndata+1,0:ndata+1) :: f
      integer :: i, j
      do i = 0, NDATA+1
         do j = 0, NDATA+1
            f(i,j) = s * f(i,j)
         end do
      end do
    end subroutine scale
```
Defines:
scale, used in chunk 98.

99a ⟨*Calculate* ∇f 99a⟩≡
```
print *, "ERROR: $\nabla f$ n.a."
stop
```
Log-likelihood won't fly, because we can't normalize the likelihood function for an unbounded parameter range. Let's use good ole least-squares instead.

99b ⟨*Calculate* f *(v1)* 99b⟩≡
```
f = 0d0
do i = 1, NDATA
   do j = 1, NDATA
      if (dfee(i,j) .gt. 0d0) then
         f = f + ((phie(xee(1,i,j),a) * phie(xee(2,i,j),a) &
                    - fee(i,j)) / dfee(i,j))**2
      end if
      if (dfeg(i,j) .gt. 0d0) then
         f = f + ((phie(xeg(1,i,j),a) * phig(xeg(2,i,j),a) &
                    - feg(i,j)) / dfeg(i,j))**2
      end if
      if (dfge(i,j) .gt. 0d0) then
         f = f + ((phig(xge(1,i,j),a) * phie(xge(2,i,j),a) &
                    - fge(i,j)) / dfge(i,j))**2
      end if
      if (dfgg(i,j) .gt. 0d0) then
         f = f + ((phig(xgg(1,i,j),a) * phig(xgg(2,i,j),a) &
                    - fgg(i,j)) / dfgg(i,j))**2
      end if
   end do
end do
```
Uses `phie` 100c and `phig` 100e.

99c ⟨*Local variables for* `fct` *(v1)* 95e⟩+≡
```
integer :: i, j
real(kind=double) :: delta
```

99d ⟨*Calculate* f *(v1)* 99b⟩+≡
```
if ((a(2) .le. -1d0) .or. (a(3) .le. -1d0/pwr)) then
   print *, "warning: discarding out-of-range a2/3: ", a(2), a(3)
   ⟨Give up on f 99e⟩
else
  delta = 1d0 - exp(a(1)) * beta(a(2)+1d0,a(3)+1d0/pwr) * dble(NDATA) / pwr
  if (delta .lt. 0d0) then
     print *, "warnimg: delta forced to 0 from ", delta
     delta = 0d0
  end if
```
Uses `beta` 105.

99e ⟨*Give up on* f 99e⟩≡
```
f = 1d100
```

99f ⟨*Calculate* f *(v1)* 99b⟩+≡
```
do i = 1, NDATA
   if (dfee(ndata+1,i) .gt. 0d0) then
```

```
            f = f + ((delta*phie(xee(2,ndata+1,i),a) &
                       - fee(ndata+1,i)) / dfee(ndata+1,i))**2
         end if
         if (dfeg(ndata+1,i) .gt. 0d0) then
            f = f + ((delta*phig(xeg(2,ndata+1,i),a) &
                       - feg(ndata+1,i)) / dfeg(ndata+1,i))**2
         end if
         if (dfee(i,ndata+1) .gt. 0d0) then
            f = f + ((delta*phie(xee(1,i,ndata+1),a) &
                       - fee(i,ndata+1)) / dfee(i,ndata+1))**2
         end if
         if (dfge(i,ndata+1) .gt. 0d0) then
            f = f + ((delta*phig(xge(1,i,ndata+1),a) &
                       - fge(i,ndata+1)) / dfge(i,ndata+1))**2
         end if
      end do
```
Uses `phie` 100c and `phig` 100e.

100a ⟨*Calculate f (v1)* 99b⟩+≡
```
      if (dfee(ndata+1,ndata+1) .gt. 0d0) then
         f = f + ((delta*delta &
                    - fee(ndata+1,ndata+1)) / dfee(ndata+1,ndata+1))**2
      end if
```

100b ⟨`circe1_fit.f90: public` 95a⟩+≡
```
    public :: phie
```
Uses `phie` 100c.

100c ⟨`circe1_fit.f90: subroutines` 95b⟩+≡
```
    function phie (x, a)
      real(kind=double) :: x, phie
      real(kind=double), dimension(6) :: a
      phie = exp (a(1) + a(2)*log(x) + a(3)*log(1d0-x))
    end function phie
```
Defines:
`phie`, used in chunks 99, 100b, and 103a.

100d ⟨`circe1_fit.f90: public` 95a⟩+≡
```
    public :: phig
```
Uses `phig` 100e.

100e ⟨`circe1_fit.f90: subroutines` 95b⟩+≡
```
    function phig (x, a)
      real(kind=double) :: x, phig
      real(kind=double), dimension(6) :: a
      phig = exp (a(4) + a(5)*log(x) + a(6)*log(1d0-x))
    end function phig
```
Defines:
`phig`, used in chunks 99, 100d, and 103a.

101a ⟨*Write output (v1)* 101a⟩≡

```
      a1(1) = exp(a(1)) * dble(NDATA) / pwr
      a1(2) = a(2)
      a1(3) = a(3) - 1d0 + 1d0/pwr
      a1(4) = exp(a(4)) * dble(NDATA) / pwr
      a1(5) = a(5) - 1d0 + 1d0/pwr
      a1(6) = a(6)
      open (10, file = 'Parameters')
      write (10, 1000) REV, tee / 1D32
      write (10, 1001) REV, &
            1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0), &
            a1(1), a1(2), a1(3), a1(4), a1(5), a1(6), &
            a1(4) * beta(a1(5)+1d0,a1(6)+1d0)
  1000 format ('      data xa5lum(@ENERGY@,@ACC@,', I2, ') / ', E12.5, ' /')
  1001 format ('      data (xa5(i,@ENERGY@,@ACC@,', I2 ,'),i=0,7) /', /, &
               '     $  ', 4(E12.5,', '), /, &
               '     $  ', 3(E12.5,', '), E12.5, ' /')
      close (10)
```

Uses beta 105.

101b ⟨*Local variables for* fct *(v1)* 95e⟩+≡

```
      ⟨Declare NPARAM 94a⟩
      real(kind=double), dimension(NPARAM) :: a1
      integer, parameter :: REV = 1
```

Uses NPARAM 94a.

The average elektron energy in the continuum can be calculated analytically:

$$\langle E_{e^\pm}\rangle_{\text{cont}} = E_{\text{beam}}\,\langle x_{e^\pm}\rangle_{\text{cont}} = E_{\text{beam}}\frac{\int dx\, x^{a_2}(1-x)^{a_3}x}{B(a_2,a_3)}$$
$$= E_{\text{beam}}\frac{B(a_2+1,a_3)}{B(a_2,a_3)} = E_{\text{beam}}\frac{a_2+1}{a_2+a_3+2} \quad (40)$$

101c ⟨*Write output (v1)* 101a⟩+≡

```
      delta = 1d0 - a1(1) * beta(a1(2)+1d0,a1(3)+1d0)
      print *, '< x_e > = ', delta + (1d0-delta)*(a1(2)+1d0)/(a1(2)+a1(3)+2d0)
```

Uses beta 105.

similarly:

$$\langle E_\gamma\rangle = E_{\text{beam}}\frac{a_5+1}{a_5+a_6+2} \quad (41)$$

101d ⟨*Write output (v1)* 101a⟩+≡

```
      print *, '< x_g > = ', (a1(5)+1d0)/(a1(5)+a1(6)+2d0)
```

Count the degrees of freedom in ndof:

101e ⟨*Write output (v1)* 101a⟩+≡

```
      ndof = 0
      do i = 0, ndata+1
         do j = 0, ndata+1
            if (dfee(i,j) .gt. 0d0) ndof = ndof + 1
            if (dfeg(i,j) .gt. 0d0) ndof = ndof + 1
            if (dfge(i,j) .gt. 0d0) ndof = ndof + 1
```

```
         if (dfgg(i,j) .gt. 0d0) ndof = ndof + 1
      end do
   end do
   print *, 'CHI2 = ', f / ndof
```

102a ⟨*Local variables for* `fct` *(v1)* 95e⟩+≡

```
   integer :: ndof
```

The error on the luminosity is just the (possibly rescaled) counting error:

102b ⟨*Write output (v1)* 101a⟩+≡

```
   open (10, file = 'Errors.tex')
   write (10, 1099) tee / 1d32, dtee / 1d32, dtee / 1d32
1099 format ('$', F8.2, '_{-', F4.2, '}^{+', F4.2, '}$')
```

After retrieving the error from `MINUIT`, we have to take care of the mapping of the parameters

$$a'_{1/4} = e^{a_{1/4}} B(a_{2/5} + 1, a_{3/6} + 1) N_{\text{bins}} \eta^{-1} \Longrightarrow \delta a'_{1/4} = a'_{1/4} \delta a_{1/4} \tag{42}$$

ignoring the errors in the integral (i.e. the Beta function).

102c ⟨*Write output (v1)* 101a⟩+≡

```
   call mnerrs (1,  eplus, eminus, epara, corr)
   ab = a1(1) * beta(a1(2)+1d0,a1(3)+1d0)
   write (10, 1100) ab, abs (ab*eminus), abs (ab*eplus)
1100 format ('$', F8.4, '_{-', F6.4, '}^{+', F6.4, '}$')
```

Uses `beta` 105.

102d ⟨*Local variables for* `fct` *(v1)* 95e⟩+≡

```
   real(kind=double) :: ab
```

The other mappings are even more trivial:

$$a'_{2/6} = a_{2/6} - 1 + \eta^{-1} \Longrightarrow \delta a'_{2/6} = \delta a_{2/6} \quad a'_{3/5} = a_{3/5} - 1 + \eta^{-1} \Longrightarrow \delta a'_{3/5} = \delta a_{3/5} \tag{43}$$

102e ⟨*Write output (v1)* 101a⟩+≡

```
   do i = 2, 3
      call mnerrs (i,  eplus, eminus, epara, corr)
      write (10, 1100) a1(i), abs (eminus), abs (eplus)
   end do
   call mnerrs (4,  eplus, eminus, epara, corr)
   ab = a1(4) * beta(a1(5)+1d0,a1(6)+1d0)
   write (10, 1100) ab, abs (ab*eminus), abs (ab*eplus)
   do i = 5, 6
      call mnerrs (i,  eplus, eminus, epara, corr)
      write (10, 1100) a1(i), abs (eminus), abs (eplus)
   end do
   close (10)
```

Uses `beta` 105.

102f ⟨*Local variables for* `fct` *(v1)* 95e⟩+≡

```
   real(kind=double) :: eplus, eminus, epara, corr
   integer :: n
```

103a ⟨*Write output (v1)* 101a⟩+≡
```
      do n = 1, 10
         call pslice ('ee','x',n,NDATA,xee,fee,dfee,phie,phie,a)
         call pslice ('eg','x',n,NDATA,xeg,feg,dfeg,phie,phig,a)
         call pslice ('ge','x',n,NDATA,xge,fge,dfge,phig,phie,a)
         call pslice ('gg','x',n,NDATA,xgg,fgg,dfgg,phig,phig,a)
         call pslice ('ee','y',n,NDATA,xee,fee,dfee,phie,phie,a)
         call pslice ('eg','y',n,NDATA,xeg,feg,dfeg,phie,phig,a)
         call pslice ('ge','y',n,NDATA,xge,fge,dfge,phig,phie,a)
         call pslice ('gg','y',n,NDATA,xgg,fgg,dfgg,phig,phig,a)
      end do
      call pslice ('ee','x',21,NDATA,xee,fee,dfee,phie,phie,a)
      call pslice ('eg','x',21,NDATA,xeg,feg,dfeg,phie,phig,a)
      call pslice ('ee','y',21,NDATA,xee,fee,dfee,phie,phie,a)
      call pslice ('ge','y',21,NDATA,xge,fge,dfge,phig,phie,a)
```
Uses phie 100c, phig 100e, and pslice 103d.

UNIX Fortran compiler want backslashes escaped:

103b ⟨*Write output (v1)* 101a⟩+≡
```
      open (10, file = 'Slices.mp4')
      write (10,*) "picture eslice[], gslice[];"
      do n = 1, NDATA
         write (10,*) 'eslice[', n, '] := ', &
              'btex $x_{e^\\pm} = ', xee(1,n,1), '$ etex;'
         write (10,*) 'gslice[', n, '] := ', &
              'btex $x_\\gamma = ', xgg(1,n,1), '$ etex;'
      end do
      close (10)
```

103c ⟨circe1_fit.f90: public 95a⟩+≡
```
    public :: pslice
```
Uses pslice 103d.

103d ⟨circe1_fit.f90: subroutines 95b⟩+≡
```
    subroutine pslice (pp, xy, n, ndata, x, f, df, phi1, phi2, a)
      character(len=2) :: pp
      character(len=1) :: xy
      integer :: n, ndata
      real(kind=double), dimension(2,0:ndata+1,0:ndata+1) :: x
      real(kind=double), dimension(0:ndata+1,0:ndata+1) :: f, df
      real(kind=double), dimension(6) :: a
      real(kind=double) :: z
      real(kind=double) :: phi1, phi2, d, delta, pwr
      external phi1, phi2
      integer :: i
      character(len=2) digits
      write (digits, '(I2.2)') n
      open (10, file = 'lumidiff-'//pp//xy//digits//'.dat')
      open (11, file = 'lumidiff-'//pp//xy//digits//'.fit')
      open (12, file = 'lumidiff-'//pp//xy//digits//'.chi')
      if (n .eq. ndata+1) then
```

```
         pwr = 5d0
         delta = 1d0 - exp(a(1))*beta(a(2)+1d0,a(3)+1d0/pwr) &
                      * dble(NDATA) / pwr
      else
         delta = 0
      end if
      if (xy .eq. 'x') then
         do i = 1, ndata
            if (df(n,i) .gt. 0d0) then
               if (pp(2:2) .eq. 'g') then
                  z = x(2,n,i)
               else
                  z = 1d0 - x(2,n,i)
               endif
               if (n .eq. ndata+1) then
                  d = delta*phi2(x(2,n,i),a)
               else
                  d = phi1(x(1,n,i),a)*phi2(x(2,n,i),a)
               endif
               write (10,*) z, f(n,i), df(n,i)
               write (11,*) z, d
               write (12,*) z, (f(n,i) - d) / df(n,i)
            endif
         end do
      else if (xy .eq. 'y') then
         do i = 1, ndata
            if (df(i,n) .gt. 0d0) then
               if (pp(1:1) .eq. 'g') then
                  z = x(1,i,n)
               else
                  z = 1d0 - x(1,i,n)
               endif
               if (n .eq. ndata+1) then
                  d = phi1(x(1,i,n),a)*delta
               else
                  d = phi1(x(1,i,n),a)*phi2(x(2,i,n),a)
               endif
               write (10,*) z, f(i,n), df(i,n)
               write (11,*) z, d
               write (12,*) z, (f(i,n) - d) / df(i,n)
            endif
         end do
      endif
      close (10)
      close (11)
      close (12)
    end subroutine pslice
```

Defines:
`pslice`, used in chunk 103.

Uses beta 105.

105 ⟨circe1_fit.f90: subroutines 95b⟩+≡

```
  function beta (a, b)
    real(kind=double) :: a, b, beta
    beta = exp (dlgama(a) + dlgama(b) - dlgama(a+b))
    contains
      function dlgama (x)
        real(kind=double) :: dlgama
        real(kind=double), dimension(7) :: p1, q1, p2, q2, p3, q3
        real(kind=double), dimension(5) :: c, xl
        real(kind=double) :: x, y, zero, one, two, half, ap, aq
        integer :: i
        data ZERO /0.0D0/, ONE /1.0D0/, TWO /2.0D0/, HALF /0.5D0/
        data XL /0.0D0,0.5D0,1.5D0,4.0D0,12.0D0/
        data p1 /+3.8428736567460D+0, +5.2706893753010D+1, &
                   +5.5584045723515D+1, -2.1513513573726D+2, &
                   -2.4587261722292D+2, -5.7500893603041D+1, &
             -2.3359098949513D+0/
        data q1 /+1.0000000000000D+0, +3.3733047907071D+1, &
             +1.9387784034377D+2, +3.0882954973424D+2, &
             +1.5006839064891D+2, +2.0106851344334D+1, &
             +4.5717420282503D-1/
        data p2 /+4.8740201396839D+0, +2.4884525168574D+2, &
             +2.1797366058896D+3, +3.7975124011525D+3, &
             -1.9778070769842D+3, -3.6929834005591D+3, &
             -5.6017773537804D+2/
        data q2 /+1.0000000000000D+0, +9.5099917418209D+1, &
             +1.5612045277929D+3, +7.2340087928948D+3, &
             +1.0459576594059D+4, +4.1699415153200D+3, &
             +2.7678583623804D+2/
        data p3 /-6.8806240094594D+3, -4.3069969819571D+5, &
                   -4.7504594653440D+6, -2.9423445930322D+6, &
             +3.6321804931543D+7, -3.3567782814546D+6, &
             -2.4804369488286D+7/
        data q3 /+1.0000000000000D+0, -1.4216829839651D+3, &
             -1.5552890280854D+5, -3.4152517108011D+6, &
             -2.0969623255804D+7, -3.4544175093344D+7, &
             -9.1605582863713D+6/
        data c / 1.1224921356561D-1,  7.9591692961204D-2, &
                  -1.7087794611020D-3,  9.1893853320467D-1, &
                   1.3469905627879D+0/
        if (x .le. xl(1)) then
             print *, 'ERROR: DLGAMA non positive argument: ', X
                 dlgama = zero
        end if
        if (x .le. xl(2)) then
          y = x + one
          ap = p1(1)
          aq = q1(1)
```

```
          do i = 2, 7
            ap = p1(i) + y * ap
            aq = q1(i) + y * aq
          end do
          y = - log(x) + x * ap / aq
        else if (x .le. xl(3)) then
          ap = p1(1)
          aq = q1(1)
          do i = 2, 7
             ap = p1(i) + x * ap
             aq = q1(i) + x * aq
          end do
          y = (x - one) * ap / aq
        else if (x .le. xl(4)) then
          ap = p2(1)
          aq = q2(1)
          do i = 2, 7
             ap = p2(i) + x * ap
             aq = q2(i) + x * aq
          end do
          y = (x-two) * ap / aq
        else if (x .le. xl(5)) then
          ap = p3(1)
          aq = q3(1)
          do i = 2, 7
             ap = p3(i) + x * ap
             aq = q3(i) + x * aq
          end do
          y = ap / aq
        else
         y = one / x**2
         y = (x-half) * log(x) - x + c(4) + &
             (c(1) + y * (c(2) + y * c(3))) / ((c(5) + y) * x)
        end if
        dlgama = y
      end function dlgama
  end function beta
```

Defines:
beta, used in chunks 59b, 61d, 65c, 70b, 71e, 75a, 99d, 101–103, and 108b.

106 ⟨circe1_fit.sh 106⟩≡

```
#! /bin/sh
# mode=${2-slow}
mode=${2-fast}
root=`pwd`
indir=${root}/${3-input}
tmpdir=${root}/tmp
outdir=${root}/output
acc="${1-sband350 sband500 sband800 sband1000 sband1600
```

```
            tesla350 tesla500 tesla800 tesla1000 tesla1600
            tesla350-low tesla500-low tesla800-low tesla1000-low tesla1600-low
            xband350 xband500 xband800 xband1000 xband1600}"
```

107a ⟨circe1_fit.sh 106⟩+≡

```
xmkdir () {
  for d in "$@"; do
    mkdir $d 2>/dev/null || true
  done
}
rm -fr ${tmpdir}
xmkdir ${outdir} ${tmpdir}
```

107b ⟨circe1_fit.sh 106⟩+≡

```
cd ${tmpdir}
cat /dev/null >${outdir}/Params.f90
for a in $acc; do
  case "$a" in
    *1600*) energy=TEV16;;
    *1000*) energy=TEV1;;
     *800*) energy=GEV800;;
     *500*) energy=GEV500;;
  *3[56]0*) energy=GEV350;;
     *170*) energy=GEV170;;
      *90*) energy=GEV090;;
         *) energy=GEV500;;
  esac
  cp ${indir}/${a}_${mode}/lumidiff-??.dat .
  ${root}/circe1_fit.bin
  rm -fr ${outdir}/${a}_${mode}
  mkdir ${outdir}/${a}_${mode}
  cp Slices.mp4 ${outdir}
  cp Errors.tex lumidiff-??x[0-9][0-9].??? ${outdir}/${a}_${mode}
  sed -e "s/@ENERGY@/$energy/g" \
      -e "s/@ACC@/`echo $a | tr a-z A-Z | tr -cd A-Z`/g" Parameters \
    >>${outdir}/Params.f90
done
cd ${root}
rm -fr ${tmpdir}
```

107c ⟨circe1_fit.sh 106⟩+≡

```
cat >${outdir}/Params.tex <<'END'
\begin{table}
  \begin{center}
    \renewcommand{\arraystretch}{1.3}
    \begin{tabular}{|c||c|c|c|c|}\hline
      & \texttt{SBAND} & \texttt{TESLA} & \texttt{TESLA'} & \texttt{XBAND}
      \\\hline\hline
END
```

Uses SBAND 13a, TESLA 13a, and XBAND 13a.

108a ⟨circe1_fit.sh 106⟩+≡
```
line () {
  for a in $acc; do
    case $a in
      *350* | *800* | *1000* | *1600*)
          ;;
      *)  echo -n ' & '
          sed -n $1p ${outdir}/${a}_${mode}/Errors.tex
          ;;
    esac
  done
  echo '\\\hline'
}
(echo '$\mathcal{L}/\text{fb}^{-1}\upsilon^{-1}$'; line 1
 echo '$\int d_{e^\pm}$';                           line 2
 echo '$x_{e^\pm}^\alpha$';                         line 3
 echo '$(1-x_{e^\pm})^\alpha$';                     line 4
 echo '$\int d_\gamma$';                            line 5
 echo '$x_\gamma^\alpha$';                          line 6
 echo '$(1-x_\gamma)^\alpha$';                      line 7
) >>${outdir}/Params.tex
```

108b ⟨circe1_fit.sh 106⟩+≡
```
cat >>${outdir}/Params.tex <<'END'
    \end{tabular}
  \end{center}
  \caption{\label{tab:param}%
    Version 1, revision 1997 04 16 of the beam spectra at 500 GeV.
    The rows correspond to the luminosity per effective year, the
    integral over the continuum and the powers in the factorized Beta
    distributions~(\ref{eq:beta}).}
\end{table}
END
```
Uses beta 105.

108c ⟨circe1_fit.sh 106⟩+≡
```
cat >>${outdir}/Params.tex <<'END'
\begin{table}
  \begin{center}
    \renewcommand{\arraystretch}{1.3}
    \begin{tabular}{|c||c|c|c|c|}\hline
      & \texttt{SBAND} & \texttt{TESLA} & \texttt{TESLA'} & \texttt{XBAND}
      \\\hline\hline
END
```
Uses SBAND 13a, TESLA 13a, and XBAND 13a.

108d ⟨circe1_fit.sh 106⟩+≡
```
line () {
  for a in $acc; do
    case $a in
      *1000*)
```

```
        echo -n ' & '
        sed -n $1p ${outdir}/${a}_${mode}/Errors.tex
        ;;
    esac
  done
  echo '\\\hline'
}
(echo '$\mathcal{L}/\text{fb}^{-1}\upsilon^{-1}$'; line 1
 echo '$\int d_{e^\pm}$';                          line 2
 echo '$x_{e^\pm}^\alpha$';                        line 3
 echo '$(1-x_{e^\pm})^\alpha$';                    line 4
 echo '$\int d_\gamma$';                           line 5
 echo '$x_\gamma^\alpha$';                         line 6
 echo '$(1-x_\gamma)^\alpha$';                     line 7
) >>${outdir}/Params.tex
```

109a ⟨circe1_fit.sh 106⟩+≡

```
cat >>${outdir}/Params.tex <<'END'
    \end{tabular}
  \end{center}
  \caption{\label{tab:param/TeV}%
    Version 1, revision 1997 04 17 of the beam spectra at 1 TeV.}
\end{table}
END
```

109b ⟨circe1_fit.sh 106⟩+≡

```
cat >>${outdir}/Params.tex <<'END'
\begin{table}
  \begin{center}
    \renewcommand{\arraystretch}{1.3}
    \begin{tabular}{|c||c|c|c|c|}\hline
      & 350 GeV & 500 GeV & 800 GeV & 1600 GeV
      \\\hline\hline
END
```

109c ⟨circe1_fit.sh 106⟩+≡

```
line () {
  for a in $acc; do
    case $a in
      tesla*-low)
        ;;
      tesla1000)
        ;;
      tesla*)
        echo -n ' & '
        sed -n $1p ${outdir}/${a}_${mode}/Errors.tex
        ;;
    esac
  done
  echo '\\\hline'
}
```

```
(echo '$\mathcal{L}/\text{fb}^{-1}\upsilon^{-1}$'; line 1
 echo '$\int d_{e^\pm}$';                         line 2
 echo '$x_{e^\pm}^\alpha$';                       line 3
 echo '$(1-x_{e^\pm})^\alpha$';                   line 4
 echo '$\int d_\gamma$';                          line 5
 echo '$x_\gamma^\alpha$';                        line 6
 echo '$(1-x_\gamma)^\alpha$';                    line 7
) >>${outdir}/Params.tex
```

110a ⟨circe1_fit.sh 106⟩+≡

```
cat >>${outdir}/Params.tex <<'END'
    \end{tabular}
  \end{center}
  \caption{\label{tab:param/Tesla}%
    Version 1, revision 1997 04 17 of the beam spectra for TESLA.}
\end{table}
END
exit 0
```

Uses TESLA 13a.

7.2 Experimental

7.2.1 Quasi One Dimensional

110b ⟨circe1_minuit1.f90 110b⟩≡

```
! circe1_minuit1.f90 -- fitting for circe
⟨Copyleft notice 29b⟩
```

Uses circe 31b.

We're utilizing the familiar "MINUIT" package [15].

110c ⟨circe1_minuit1.f90 110b⟩+≡

```
⟨Minuit1 module 110d⟩
⟨Minuit1 main program 111a⟩
```

110d ⟨Minuit1 module 110d⟩≡

```
module minuit1
  use kinds

  implicit none

  public :: fct
  public :: phi

contains

⟨Function to minimize 111c⟩

⟨Function phi1 112d⟩
end module minuit1
```

Defines:
minuit1, used in chunk 111a.
Uses fct 95b 111c and phi 112d 116.

111a ⟨*Minuit1 main program* 111a⟩≡
```
program fit
  use kinds
  use minuit1

  implicit none

  call minuit (fct, 0d0)
end program fit
```
Uses fct 95b 111c, fit 93b, and minuit1 110d.

111b ⟨*Minuit2 main program* 111b⟩≡
```
program fit
  use kinds
  use minuit2

  implicit none

  call minuit (fct, 0d0)
end program fit
```
Uses fct 95b 111c, fit 93b, and minuit2 115f.

111c ⟨*Function to minimize* 111c⟩≡
```
  subroutine fct (nx, df, f, a, mode, g)
    integer, intent(in) ::  nx, mode
    real(kind=double) :: f, g
    real(kind=double), dimension(:) :: df, a
    ⟨Local variables for fct 112a⟩
    if (mode .eq. 1) then
      ⟨Read input data 111d⟩
    else if (mode .eq. 2) then
      ⟨Calculate ∇f 99a⟩
    end if
    ⟨Calculate f 112b⟩
    if (mode .eq. 3) then
       ⟨Write output 112c⟩
    end if
  end subroutine fct
```

Defines:
fct, used in chunks 93b, 95a, 110, 111, and 115f.

111d ⟨*Read input data* 111d⟩≡
```
    open (10, file = 'minuit.data')
    do i = 1, NDATA
       do j = 1, NDATA
          read (10, *) xi(1,i,j), xi(2,i,j), fi(i,j), dfi(i,j)
          fi(i,j) = fi(i,j)/1d30
```

```
      dfi(i,j) = dfi(i,j)/1d30
    end do
  end do
  close (10)
```

112a ⟨*Local variables for* `fct` 112a⟩≡

```
  integer, parameter :: NDATA = 20
  real(kind=double) :: chi, chi2
  real(kind=double), dimension(2,NDATA,NDATA) :: xi
  real(kind=double), dimension(NDATA,NDATA) :: fi, dfi
  integer :: i, j, n
```

112b ⟨*Calculate f* 112b⟩≡

```
  f = 0d0
  do i = 1, NDATA
    do j = 1, NDATA
      if (dfi(i,j).gt.0d0) then
        f = f + ((phi(xi(1,i,j),xi(2,i,j),a) &
                  - fi(i,j)) / dfi(i,j))**2
      end if
    end do
  end do
```

Uses phi 112d 116.

112c ⟨*Write output* 112c⟩≡

```
  chi2 = 0d0
  n = 0
  open (10, file = 'minuit.fit')
  do i = 1, NDATA
    do j = 1, NDATA
      if (dfi(i,j).gt.0d0) then
        chi = (phi(xi(1,i,j),xi(2,i,j),a)-fi(i,j))/dfi(i,j)
        write (10,*) xi(1,i,j), xi(2,i,j), &
                     1d30 * phi(xi(1,i,j),xi(2,i,j),a), &
                     1d30 * fi(i,j), &
                     chi
        chi2 = chi2 + chi**2
        n = n + 1
      else
        write (10,*) xi(1,i,j), xi(2,i,j), &
                     1d30 * phi(xi(1,i,j),xi(2,i,j),a), &
                     1d30 * fi(i,j)
      end if
    end do
  end do
  close (10)
  print *, 'CHI2 = ', chi2/n
```

Uses phi 112d 116.

112d ⟨*Function phi1* 112d⟩≡

```
function phi (e1, e2, a)
  real(kind=double) :: e1, e2
```

```
real(kind=double), dimension(17) :: a
real(kind=double) :: phi
real(kind=double) :: y1, y2
y1 = e1 / 250d0
y2 = e2 / 250d0
phi = exp (                           &
     + a( 1) * 1d0                    &
     + a( 2) * log(y1)                &
     + a( 3) * log(1d0-y1)            &
     + a( 4) * log(-log(y1))          &
     + a( 5) * log(-log(1d0-y1))      &
     + a( 6) * y1                     &
     + a( 7) * log(y1)**2             &
     + a( 8) * log(1d0-y1)**2         &
     + a( 9) * log(-log(y1))**2       &
     + a(10) * log(-log(1d0-y1))**2   &
     + a(11) * y1**2                  &
     + a(12) / log(y1)                &
     + a(13) / log(1d0-y1)            &
     + a(14) / log(-log(y1))          &
     + a(15) / log(-log(1d0-y1))      &
     + a(16) / y1                     &
     + a(17) / (1d0-y1)               &
     + a( 2) * log(y2)                &
     + a( 3) * log(1d0-y2)            &
     + a( 4) * log(-log(y2))          &
     + a( 5) * log(-log(1d0-y2))      &
     + a( 6) * y2                     &
     + a( 7) * log(y2)**2             &
     + a( 8) * log(1d0-y2)**2         &
     + a( 9) * log(-log(y2))**2       &
     + a(10) * log(-log(1d0-y2))**2   &
     + a(11) * y2**2                  &
     + a(12) / log(y2)                &
     + a(13) / log(1d0-y2)            &
     + a(14) / log(-log(y2))          &
     + a(15) / log(-log(1d0-y2))      &
     + a(16) / y2                     &
     + a(17) / (1d0-y2)               &
     )
end function phi
```

Defines:
`phi`, used in chunks 110d, 112, and 115f.

113 ⟨circe1_minuit1.sh 113⟩≡

```
#! /bin/sh
minuit_bin=`pwd`/circe1_minuit1.bin
⟨Process arguments 114a⟩
(
```

```
    ⟨Define parameters 114d⟩
    ⟨Fix parameters 114e⟩
    ⟨Fix strategy 115a⟩
    ⟨Run Minuit 115b⟩
  ) | eval "$minuit_bin $filter"
  ⟨Maybe plot results 115c⟩
  exit 0
```

114a ⟨*Process arguments* 114a⟩≡

```
tmp="$IFS"
  IFS=:
  args=":$*:"
IFS="$tmp"
```

114b ⟨*Process arguments* 114a⟩+≡

```
filter="| \
  awk '/STATUS=(CONVERGED|CALL LIMIT|FAILED)/ { p=1; print }; \
       /@.* \.00000 *fixed/ { next }; \
       /EDM=|CHI2|@/ && p { print }' "
```

114c ⟨*Process arguments* 114a⟩+≡

```
case "$args" in
  *:v:*) filter=;;
esac
```

114d ⟨*Define parameters* 114d⟩≡

```
cat <<END
set title
CIRCE
parameters
1  '@ 1         ' 0.00 0.01
2  '@ lx        ' 0.20 0.01
3  '@ l(1-x)    ' 0.20 0.01
4  '@ llx       ' 0.00 0.01
5  '@ ll(1-x)   ' 0.00 0.01
6  '@ x         ' 0.00 0.01
7  '@ lx^2      ' 0.00 0.01
8  '@ l(1-x)^2  ' 0.00 0.01
9  '@ llx^2     ' 0.00 0.01
10 '@ ll(1-x)^2' 0.00 0.01
11 '@ x^2       ' 0.00 0.01
12 '@ 1/lx      ' 0.00 0.01
13 '@ 1/l(1-x)  ' 0.00 0.01
14 '@ 1/llx     ' 0.00 0.01
15 '@ 1/ll(1-x)' 0.00 0.01
16 '@ 1/x       ' 0.00 0.01
17 '@ 1/(1-x)   ' 0.00 0.01

END
```

114e ⟨*Fix parameters* 114e⟩≡

```
for p in 1  2  3  4  5  6  7  8  9 10 \
```

```
          11 12 13 14 15 16 17; do
    case "$args" in
      *:$p=*:*) val=`echo "$args" | sed 's/.*:'"$p"'=\\([0-9.-]*\\):.*/\\1/'`;
                echo set parameter $p $val;
                echo fix $p;;
        *:$p:*) ;;
             *) echo fix $p;;
    esac
  done
```

115a ⟨*Fix strategy* 115a⟩≡

```
  case "$args" in
    *:S0:*) echo set strategy 0;;
    *:S1:*) echo set strategy 1;;
    *:S2:*) echo set strategy 2;;
  esac
```

115b ⟨*Run Minuit* 115b⟩≡

```
  cat <<END
  migrat 10000 0.01
  stop
  END
```

115c ⟨*Maybe plot results* 115c⟩≡

```
  case "$args" in
    *:p:*) awk '$5 != "" { print $1, $2, $5 }' minuit.fit > chi2
           awk '$5 != "" { print $1, $5 }' minuit.fit > chix
           awk '$5 != "" { print $2, $5 }' minuit.fit > chiy
           gnuplot -geometry -0+0 plot2 >/dev/null 2>&1
  esac
```

Uses `fit` 93b.

7.2.2 Quasi Two Dimensional

115d ⟨`circe1_minuit2.f90` 115d⟩≡

```
  ! minuit2.f90 -- fitting for circe
  ⟨Copyleft notice 29b⟩
```

Uses `circe` 31b and `minuit2` 115f.

115e ⟨`circe1_minuit2.f90` 115d⟩+≡

```
  ⟨Minuit2 module 115f⟩
  ⟨Minuit2 main program 111b⟩
```

115f ⟨*Minuit2 module* 115f⟩≡

```
  module minuit2
    use kinds

    implicit none

    public :: fct
    public :: phi
```

```
contains

⟨Function to minimize 111c⟩
⟨Function phi2 116⟩
end module minuit2
```

Defines:
`minuit2`, used in chunks 111b and 115d.
Uses `fct` 95b 111c and `phi` 112d 116.

116 ⟨*Function phi2* 116⟩≡

```
  function phi (e1, e2, a)
    real(kind=double) :: e1, e2
    real(kind=double), dimension(33) :: a
    real(kind=double) :: phi
    real(kind=double) :: y1, y2
    y1 = e1 / 250d0
    y2 = e2 / 250d0
    phi = exp (                          &
         + a( 1) * 1d0                   &
         + a( 2) * log(y1)               &
         + a( 3) * log(1d0-y1)           &
         + a( 4) * log(-log(y1))         &
         + a( 5) * log(-log(1d0-y1))     &
         + a( 6) * y1                    &
         + a( 7) * log(y1)**2            &
         + a( 8) * log(1d0-y1)**2        &
         + a( 9) * log(-log(y1))**2      &
         + a(10) * log(-log(1d0-y1))**2  &
         + a(11) * y1**2                 &
         + a(12) / log(y1)               &
         + a(13) / log(1d0-y1)           &
         + a(14) / log(-log(y1))         &
         + a(15) / log(-log(1d0-y1))     &
         + a(16) / y1                    &
         + a(17) / (1d0-y1)              &
         + a(18) * log(y2)               &
         + a(19) * log(1d0-y2)           &
         + a(20) * log(-log(y2))         &
         + a(21) * log(-log(1d0-y2))     &
         + a(22) * y2                    &
         + a(23) * log(y2)**2            &
         + a(24) * log(1d0-y2)**2        &
         + a(25) * log(-log(y2))**2      &
         + a(26) * log(-log(1d0-y2))**2  &
         + a(27) * y2**2                 &
         + a(28) / log(y2)               &
         + a(29) / log(1d0-y2)           &
         + a(30) / log(-log(y2))         &
         + a(31) / log(-log(1d0-y2))     &
         + a(32) / y2                    &
```

```
        + a(33) / (1d0-y2)                &
        )
    end function phi
```

Defines:
`phi`, used in chunks 110d, 112, and 115f.

117a ⟨`circe1_minuit2.sh` 117a⟩≡

```
#! /bin/sh
minuit_bin=`pwd`/circe1_minuit2.bin
⟨Process arguments 114a⟩
(
  ⟨Define parameters (2dim) 117b⟩
  ⟨Fix parameters (2dim) 118⟩
  ⟨Fix strategy 115a⟩
  ⟨Run Minuit 115b⟩
) | eval "$minuit_bin $filter"
⟨Maybe plot results 115c⟩
exit 0
```

117b ⟨*Define parameters (2dim)* 117b⟩≡

```
cat <<END
set title
CIRCE
parameters
1  '@ 1         ' 0.00 0.01
2  '@ lx        ' 0.20 0.01
3  '@ l(1-x)    ' 0.20 0.01
4  '@ llx       ' 0.00 0.01
5  '@ ll(1-x)   ' 0.00 0.01
6  '@ x         ' 0.00 0.01
7  '@ lx^2      ' 0.00 0.01
8  '@ l(1-x)^2  ' 0.00 0.01
9  '@ llx^2     ' 0.00 0.01
10 '@ ll(1-x)^2' 0.00 0.01
11 '@ x^2       ' 0.00 0.01
12 '@ 1/lx      ' 0.00 0.01
13 '@ 1/l(1-x)  ' 0.00 0.01
14 '@ 1/llx     ' 0.00 0.01
15 '@ 1/ll(1-x)' 0.00 0.01
16 '@ 1/x       ' 0.00 0.01
17 '@ 1/(1-x)   ' 0.00 0.01
18 '@ ly        ' 0.20 0.01
19 '@ l(1-y)    ' 0.20 0.01
20 '@ lly       ' 0.00 0.01
21 '@ ll(1-y)   ' 0.00 0.01
22 '@ y         ' 0.00 0.01
23 '@ ly^2      ' 0.00 0.01
24 '@ l(1-y)^2  ' 0.00 0.01
25 '@ lly^2     ' 0.00 0.01
26 '@ ll(1-y)^2' 0.00 0.01
```

```
27 '@ y^2      ' 0.00 0.01
28 '@ 1/ly     ' 0.00 0.01
29 '@ 1/l(1-y) ' 0.00 0.01
30 '@ 1/lly    ' 0.00 0.01
31 '@ 1/ll(1-y)' 0.00 0.01
32 '@ 1/y      ' 0.00 0.01
33 '@ 1/(1-y)  ' 0.00 0.01

END
```

118 ⟨*Fix parameters (2dim)* 118⟩≡

```
for p in 1  2  3  4  5  6  7  8  9 10 \
        11 12 13 14 15 16 17 18 19 20 \
        21 22 23 24 25 26 27 28 29 30 \
        31 32 33; do
  case "$args" in
    *:$p=*:*) val=`echo "$args" | sed 's/.*:'"$p"'=\\([0-9.-]*\\):.*/\\1/'`;
              echo set parameter $p $val;
              echo fix $p;;
      *:$p:*) ;;
           *) echo fix $p;;
  esac
done
```

7.3 Version 2

8 Conclusions

I have presented a library of simple parameterizations of realistic $e^{\pm}$- and γ-beam spectra at future linear $e^{+}e^{-}$-colliders. The library can be used for integration and event generation. Emphasis is put on simplicity and reproducibility of the parameterizations for supporting reproducible physics simulations.

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Daniel Schulte made his simulation code `Guinea-Pig` available and answered questions. Harald Anlauf and Torbjörn Sjöstrand have contributed useful suggestions. The Tesla group at DESY/Zeuthen made error estimates feasible by donating time on the multi-headed number cruncher `Hydra`. The 1996 ECFA/Desy Linear Collider Workshop got me started and provided support. Thanks to all of them.

Identifiers

Refinements

Index

References

[1] H. Murayama and M. E. Peskin, SLAC-PUB-7149, to appear in *Ann. Rev. Nucl. Part. Sci.*; P. Zerwas, DESY 94-001-REV.

[2] P. Chen and R. J. Noble, SLAC-PUB-4050; M. Bell and J. S. Bell, Part. Accl. **24**, 1 (1988); R. Blankenbecler and S. D. Drell, Phys. Rev. Lett. **61**, 2324 (1988); P. Chen and K. Yokoya, Phys. Rev. Lett. **61**, 1101 (1988); M. Jacob and T. T. Wu, Nucl. Phys. **B303**, 389 (1988); V. N. Baier, V. M. Katkov, and V. M. Strakovenkov, Nucl. Phys. **B328**, 387 (1989); R. Blankenbecler, S. D. Drell, and N. Kroll, Phys. Rev. **D40**, 2462 (1989); P. Chen and V. L. Telnov, Phys. Rev. Lett. **63**, 1796 (1989).

[3] K. Yokoya, KEK 85-9, KEK.

[4] P. Chen *et al.*, Nucl. Inst. Meth. **A355**, 107 (1995).

[5] D. Schulte, Ph.D. thesis, in preparation.

[6] R. B. Palmer, Ann. Rev. Nucl. Part. Sci. **40**, 529 (1990).

[7] P. Chen, Phys. Rev. **D46**, 1186 (1992).

[8] Tesla Collaboration, Conceptual Design Report, in preparation.

[9] Desy-Darmstadt Linear Collider Collaboration, Conceptual Design Report, in preparation.

[10] JLC Group, KEK Report 92-16.

[11] NLC ZDR Design Group, SLAC-Report-474.

[12] NLC ZDR Design Group and NLC Physics Working Groups, SLAC-Report-485.

[13] Particle Data Group, Phys. Rev. **D50**, 1173 (1994).

[14] G. Altarelli, R. Kleiss, and C. Verzegnassi, CERN Yellow Report 89-08.

[15] F. James and M. Roos, *MINUIT, Function Minimization and Error Analysis, Release 89.12j*, CERN, Geneva, 1989.

[16] H. Anlauf, IKDA 96/6.

[17] H. Anlauf, private communication.

[18] A. Atkinson and J. Whittaker, Appl. Stat. **28**, 90 (1979).

[19] D. E. Knuth, *Literate Programming*, Vol. 27 of *CSLI Lecture Notes* (Center for the Study of Language and Information, Leland Stanford Junior University, Stanford, CA, 1991).

[20] D. E. Knuth, *TEX: The Program*, Vol. B of *Computers & Typesetting* (Addison-Wesley, Reading, Mass., 1986).

[21] D. E. Knuth, *METAFONT: The Program*, Vol. D of *Computers & Typesetting* (Addison-Wesley, Reading, Mass., 1986).

[22] N. Ramsey, IEEE Software **11**, 97 (1994).

A Literate Programming

A.1 Paradigm

I have presented the sample code in this paper using the *literate programming* paradigm. This paradigm has been introduced by Donald Knuth [19] and his programs TeX [20] and METAFONT [21] provide excellent examples of the virtues of literate programming. Knuth summarized his intention as follows ([19], p. 99)

> "Let us change our traditional attitude to the construction of programs. Instead of imagining that our main task is to instruct a *computer* what to do, let us concentrate rather on explaining to *human beings* what we want a computer to do."

Usually, literate programming uses two utility programs to produce two kinds of files from the source

`tangle` produces the computer program that is acceptable to an "illiterate" (Fortran, C, etc.) compiler. This process consists of stripping documentation and reordering code. Therefore it frees the author from having to present the code in the particular order enforced by a compiler for purely technical reasons. Instead, the author can present the code in the order that is most comprehensible.

`weave` produces a documents that describes the program. Extensive cross referencing of the code sections is usually provided, which has been suppressed in this paper. If a powerful typesetting system (such a TeX) is used, the document can present the algorithms in clear mathematical notation alongside the code. These features improve readability and maintainability of scientific code immensely.

A.2 Practice

`Circe1` uses the `noweb` [22] system. This system has the advantage to work with any traditional programming language and support the essential features described in section A.1 with minimal effort. `noweb`'s `tangle` program only reorders the code sections, but does not reformat them. Therefore its output can be used just like any other "illiterate" program.

The examples above should be almost self-explaining, but in order to avoid any ambiguities, I give another example:

125a ⟨*Literate programming example* 125a⟩≡
  ⟨*Code that has to be at the top* 125c⟩
  ⟨*Other code* 125b⟩

I can start the presentation with the first line of the "other code":

125b ⟨*Other code* 125b⟩≡
```
line 1 of the other code
```

If appropriate, the first line of the code that has to appear *before* the other code can be presented later:

125c ⟨*Code that has to be at the top* 125c⟩≡
```
line 1 of the code at the top
```

Now I can augment the sections:

126a ⟨*Other code* 125b⟩+≡
```
line 2 of the other code
```

126b ⟨*Code that has to be at the top* 125c⟩+≡
```
line 2 of the code at the top
```

The complete "program" will be presented to the compiler as

```
line 1 of the code at the top
line 2 of the code at the top
line 1 of the other code
line 2 of the other code
```

The examples in section 3.1.1 show that this reordering is particularly useful for declaring variables when they are first used (rather than at the beginning) and for zooming in on code inside of loops.

B Fortran Name Space

In addition to the ten procedures and one `common` block discussed in section 3

- `circe`, `circee`, `circeg`, `circgg`,
- `girce`, `gircee`, `girceg`, `gircgg`,
- `circes`, `circel`, `/circom/`,

there are two more globally visible functions which are used internally:

- `circem`: error message handler,
- `girceb`: efficient Beta distribution generator.

Even if the `/circom/` is globally visible, application programs *must not* manipulate it directly. The `circes`, subroutine is provided for this purpose and updates some internal parameters as well.

With features from the current Fortran standard (Fortran90), I could have kept the last two functions and the `common` block private.

Application programs wishing to remain compatible with future versions of `Circe1` must not use `common` blocks or procedures starting with `circe` or `girce`.

C Updates

Information about updates can be obtained

- on the World Wide Web:

  `http:http://projects.hepforge.org/whizard/`

Contributions of results from other simulation programs and updated accelerator designs are welcome at

`ohl@physik.uni-wuerzburg.de`